An Incremental Cost Model of Message Toll Telephone Services

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M. D. Godfrey

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PREFACE to 2018 Edition

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The purpose of this new edition is to provide a more readable online copy. This edition is intended to be exactly as the original in content. However, the text has been OCR'ed and converted to TEX. Thus, this online PDF is, it is hoped, more readable. And, hyperlinks make locating parts of the text much easier. The Figures and Tables from the original have simply been scanned to JPEG format. Several "inserts" have also been scanned as they contained layouts which would have been difficult to reproduce in TEX.

In this edition it has been intended to exactly preserve all of the original text, figures, and tables. This includes the Proprietary Notice at the front. When we considered this publication it was agreed by AT&T that it would be published. However, near the end of the work and after the publisher had been selected, AT&T decided that publication for outside distribution would not be allowed. And, the publisher's name does not appear in the book. In any case, a substantial number of copies were printed and were used within the Bell System as part of the documentation of the system which was extensively used. And, this use of the system continued even after divestiture within, in particular, the Long Lines organization. After the divestiture I spoke with Bob Breedlove about distribution of this publication outside the former AT&T System. He said that there should be no remaining restriction. Very sadly, Bob died many years ago.

A key result of this work was a fundamental change in the relationship between AT&T and the FCC. Very briefly: The consent decree of 1956 placed AT&T under regulation by the FCC. The basic intent was to prevent unfair monopolistic behavior. Two key rules were: 1) No "cross-subsidies" would be allowed. This was to prevent AT&T from unfairly overcharging some customers and undercharging others in order to enhance their ability to gain advantages over competitors. And 2) AT&T was required to set prices so that they met, at most, a "fair" rate of return on their invested capital (termed the "rate base"). Periodically, AT&T was required to file a plan (the "rate plan") with the FCC which set the prices for their services and specified the planned investments to support the resulting demand. This was expected to satisfy the FCC rules. An unfortunate result appeared quite quickly: the achieved results did not match up with the intended results as expressed in the rate plan. No one felt that this was a satisfactory outcome. AT&T found that their expectations did not materialize due to many factors. While the FCC felt suspicious that AT&T was not acting in an appropriate manner. The evidence was that, as often happens in business, the estimates of the effects of investment and pricing policies did not produce the intended results.

Thus, the intent of the work described here was to resolve this issue by developing a methodology which would permit price and resource determination which would meet the FCC rules and which would accurately match the achieved results. This was achieved from the first filing which was based on the methods described here. After a fairly short period the FCC recognized that there had been a fundamental change. AT&T was fairly open in describing their methodology and a more positive relationship developed. A high point came when the FCC technical staff asked for a copy of our tools in order that they could better understand the process. AT&T agreed and the modeling tools and documentation were provided to the FCC. In order to make use of the tools, computing systems like those used at AT&T would be needed. The FCC simply placed an order for a computer system "exactly like the one in use at AT&T". They could then not only satisfy themselves about the assumptions and results, but could also determine the outcomes of other choices. At this point there was a firm basis for cooperation. This continued until the conclusion of the "Computer Inquiry II" (Wiki at: wikisource.org) Congressional investigation which was undertaken to examine the extent to which AT&T might be misusing its monopolistic power, particularly in the development of computer systems. This resulted in the 1980 divestiture of the various AT&T companies into separate entities. However, the separate AT&T Long Lines Company continued to use the system as described here.

A key fact that is only implied in the text is that AT&T had, from its origins, maintained detailed and accurate records of all its facilities and installed equipment. This information was essential to our analysis. The collection and processing of all the facilities data along with the detailed facilities usage required extensive work and active support throughout the Company. The data were provided from all Operating Companies to the Corporate Data Center in Piscataway, New Jersey and then transferred to the AT&T Corporate Center at John St. New York where the analysis which is described here was performed. This analysis was, at the time, described as the largest data analysis project to be undertaken.

This first printing may well contain typographical errors introduced during the conversion to the present form. I have proofread the text more than once, but no proofreading finds all the errors. In any case the original scanned copy is always available on the Wayback Machine at: archive.org. This original version appears to have contained very few typographical errors. This is surely due to the meticulous editing by Shirley Rokos and, of course, by Bob Breedlove.

I will provide updated printings if needed.

Finally, I wish to express my deep sense of privilege to have been given the opportunity to contribute to this effort. The commitment by AT&T to undertake this effort was quite exceptional. The full support of Bob Breedlove, Martin Wilk and John Tukey, as well as that of the very large group of Bell System staff made the work not only possible but a joy to experience.

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An Incremental Cost Model of Message Toll Telephone Services

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American Telephone and Telegraph Company
195 Broadway
New York City, New York
1975

Proprietary Notice

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FOREWORD

During the Bell System's interstate rate filing in Docket 16258 before the Federal Communications Commission, long-run incremental costs were proposed by AT&T as an appropriate basis for service pricing. The proposal was contingent upon the assumption that studies would be undertaken in the Company in order to develop appropriate economic theory and procedures for estimating or calculating long-run incremental costs, and was based on preliminary investigations and analyses in the area.

Actual planning for an incremental cost study at AT&T began in 1968, and the work reported here took place during the period from the onset of activity through the end of 1970. Subsequent developments have naturally resulted in modifications to the work as described, but the fundamental design and its underlying theory remain virtually as they were originally conceived. Proprietorship and continuing research of the model have been transferred from AT&T's General Departments at Corporate Headquarters to the AT&T Long Lines Department, together with some of the personnel originally associated with the computing aspects of the project in order to ensure a continuum in development.

At the outset, in 1968, Messrs. A. Froggatt and J. H. Miller of AT&T's Engineering Economics area supported the organization of a group of people to form the core of a Demand-Cost Study Team incorporating both guidance and research capabilities. The group which evolved can best be described as a cooperative coalition of managers at many levels, from not only AT&T but Bell Telephone Laboratories (BTL) and AT&T Long Lines as well. At the head of this team was Mr. A. C. Ocker, a Director in the Engineering Economics Division of AT&T. Mr. Ocker was assigned overall project responsibility, and in addition oversaw the collection of a large portion of the necessary data for the study.

Under Mr. Ocker, two guidance committees were set up. One was organized to deal with the demand, or market, aspects of the project, and the other to deal with the supply, or costs, side. Each consisted of members of AT&T, BTL, and AT&T Long Lines, all of whom acted as consultants to the personnel assigned to develop analyses in specific modules within the study.

The responsibility for assigning and overseeing the analytical modeling necessary for the project was given to the Management Sciences Division (MSD), which was then in the Comptroller's Department at AT&T. Mr. H. M. Boettinger, head of the Division, provided essential support and encouragement through all phases of this work. Mr. H. J. McMains, a Director in that organization, coordinated the activity, while Mr. M. B. Wilk, who was then Statistical Director, Bell Telephone Labs, and myself co-chaired the analytical modeling efforts. Mr. McMains's support and participation were critical to the project's success, and his insight proved especially helpful during the formative stages of the work. Mr. Wilk, in addition to his role as co-leader, was instrumental in initiating the study, in the overall design of the model, and in the final implementation of analyses. Mr. Wilk and I are presently Directors in the AT&T Corporate Planning Division.

In addition to the co-workers already named, I am indebted to many others who contributed freely of their time and expertise during various stages of the study. Together with Messrs. Ocker and Mc Mains, the Guidance Committee consisted of Messrs. A. Opie, C. McCarthy, and A. McKeage of AT&T; J. Weber of BTL; and R. Auray of AT&T Long Lines. These people provided valuable assistance by coordinating the contributions of people employed in the three companies, as well as giving advice and assistance at various stages.

Within each module of activity, there were many participants. Although the list of active project members grew exceedingly long at times, I feel compelled to try to thank personally all of those whose names were recorded in the project documentation, so that the reader will know where the true credit belongs. In so doing, I am aware of the risk that I may inadvertently omit someone, and this is cause for concern; I therefore extend to anyone overlooked my sincere apologies, and at the same time my deep gratitude for their efforts.

The module leaders were:

A. McKeage—AT&T	Demand
A. Opie—AT&T	Rates
R. Jones—AT&T	Data Management, Facilities Requirements
J. Weber—BTL	Facilities and Engineering Principles
J. Henehan—AT&T	Cost and Expense Functions
E. Fuchs—AT&T	Programming
M. Godfrey—BTL	Computer Systems and Programming
T. J. Stewart—AT&T	Computer Systems and Programming
E. Smith—AT&T	Documentation

The role of each of these individuals was vital, and each was an active participant at various periods. One group, in particular, whose contribution can clearly be seen in retrospect to have been essential to the project's ultimate success was the small but dedicated and highly talented programming staff under the direction of T. J. Stewart. This group of experts, who were given the highest quality of facilities available to carry out their responsibilities, produced a truly remarkable result as the descriptions which follow attest. There is undoubtedly more to be learned from the principles applied in the computing aspects of the work than there is from the particulars of design or equipment choices made, vulnerable as those choices are to obsolescence unless frequently reviewed and examined. Guiding principles such as those to which the programming staff adhered tend to be relatively stable over time, and it is therefore not unreasonable to hope that the project's contribution to the art of analytical modeling will endure. I would like to thank Mr. B. E. Davis, who was a member of MSD, for his advice and help related to the interpretation of relevant economic concepts.

On the following pages are listed the personnel assigned to each module, together with their specific responsibilities. These are AT&T employees, rotational employees from Bell System Associated Companies, and members of the staff of Long Lines and Bell Telephone Laboratories. To all, my gratitude for the unstinting effort which allowed us all to complete an "impossible," but highly rewarding, undertaking.

This work is co-authored by Michael Godfrey, formerly at Bell Telephone Laboratories, and presently at Imperial College of Science and Technology, London, England. Mr. Godfrey worked with me closely in the computing systems, statistical, and programming aspects of the project.

Mr. Godfrey and I would like to thank Mr. J. W. Tukey, of Princeton University and Bell Telephone Labs, for his advice and encouragement which contributed substantially to our confidence and ability to undertake this research effort.

Finally, Mr. Godfrey and I would like to thank Mrs. Shirley H. Rokos of AT&T, who was formerly a member of my staff and assisted in the preparation of this book. Mrs. Rokos co-authored portions of the text, edited the manuscript through several drafts, oversaw and scheduled the work of the illustrator, and coordinated the many production tasks involved. I am deeply appreciative of her encouragement and support.

R. L. Breedlove
Director, Corporate Planning
American Telephone and Telegraph Co.
July 1975

Project Activity, 1968-1970

Modules & Sub-Modules	*Personnel Involved and General Area of Activity*
1. Demand Inputs	A. Opie—Module Leader K. Mettler—Coordination P. Boettinger, A. McKeage—Advisory
2. Forecast Models	
a. MTS	R. Auray—Module Leader G. Hammil—MTS Model Manager
b. WATS	J. Streamo—Data Collection, Analysis, and Modeling T. Thompson and E. Singletary
3. Demand Translators	
MTS and WATS	W. Larson—Modeling W. Hall—Design and Programming F. Donahue—Programming D. Crouch—Traffic Translation, Data Collection, and Analysis
4. Data Management	R. Jones—Module Leader
a. Message Data	J. McNulty—Data Collection and Processing
b. Switching Machine Profile	H. Kosow—Data Collection and Processing
c. Intertoll Circuit Inventory	W. Mitchell—Data Collection and Processing
d. Selected Switching Machine Data	H. Kosow, J. McNulty—Data Collection

5. Facilities Requirements	J. Weber, R. Jones—Module Leaders
a. Intertoll Circuits	G. Faulhaber and R. Johnson —Model Design K. Nocella—Programming
b. Toll Connecting Trunks	G. Faulhaber and R. Johnson —Model Design M. Ring, J. Newman—Programming
c. Toll Dial Switching Equipment	E. Hoover, S. Levine, and R. Hofshi —Model Design J. McNulty—Data Collection and Programming Coordination R. Gottdenker—Programming H. Kosow—Data Collection and Model Design
d. Switching Equipment, Manual	F. Reddan—Data Collection E. Hoover, S. Levine, D. Hudson, and R. Carlson—Modeling
e. Special Equipment at Switchers	J. Henehan and R. Patterson
f. Local Switching Equipment	J. Henehan and R. Patterson
g. Exchange Plant	J. Henehan, R. Patterson, and W. Hardgrave
6. Facility Mix	
a. Intertoll Circuits	R. Genthner and C. Wagner
b. Switching Equipment	R. Patterson, E. Hoover, and J. Henehan
7. Cost Functions	J. Henehan and J. Cogswell —Module Leaders
a. Circuits	C. Wagner and J. Adams —Data Collection and Procedure Development E. Duchesne—Programming Coordinator
b. Terminals	E. Neibert, R. Field, J. Adams, C. Wagner, and D. Crouch —Data Collection and Procedure Development

c. Switching Facilities	
—Machines and Cordboards	J. Bundy—Data Collection
—AMA Equipment and all Other Switchboards	R. Patterson—Data Collection
—TSPS	J. Gabbe—Model Design
—Land and Buildings	D. Crouch—Data Collection and Analysis
8. Expense Functions	J. Henehan—Module Leader
a. Testing	R. Genthner and F. Anker —Data Collection E. Duchesne—Programming Coordinator
b. Switching Maintenance	J. Bundy—Data Collection B. Murphy and M. Torrey —Model Design and Analysis
b. Circuit Maintenance	R. Genthner—Data Collection E. Duchesne—Programming Coordinator J. Gabbe—Sample Design, Statistical Analysis, and Model Design
d. Traffic	F. Reddan—Data Collect on R. Carlson and D. Hudsoin —Sample Design, Statistical Analysis, and Model Design
e. Commercial	
—Business Office	S. Cebulski—Data Collection
—Public Telephone Commissions	W. Williams and A. Chaudry —Statistical Analysis and Model Design
—Directory	R. Patterson
f. Revenue Accounting	S. Cebulski—Data Collection A. Chaudry—Statistical Analysis and Model Design
g. General (Legal, Treasury, Engineering, Executive, Advertising)	R. Genthner—Expense Development

9. Separations	J. Henehan—Module Leader
a. Toll Connecting and Intertoll Circuits Toll Switching Local Switching	R. Patterson—Data Collection, Coordination and Model Design K. Nocella—Programming
b. Exchange Plant	W. Hardgrave—Statistical Analysis and Model Design
c. State-Interstate-Local	J. Henehan and R. Patterson
10. Other	R. Genthner—Module Leader
Taxes Other Rate Base Items Other Investment Depreciation Interest Charged to Construction Return on Investment Investment Credit Amortization	R. Genthner and R. Patterson —Data Collection
11. Economies of Scale	M. Godfrey—Model Design
12. Computer Systems and Programming	M. Godfrey and T. Stewart— —Module Co-leaders
13. Module Interface	T. Stewart—Design and Coordination
14. Input-Output Specifications	M. Godfrey and T. Stewart —Design and Implementation Standards
15. Data Library	E. Bogart
16. System Merge	T. Stewart and M. Godfrey —Process Design

17. Statistical Analysis

a. Switching Equipment Costs Switching Facilities Mix AMA Costs Land and Building Costs	J. Gabbe—Module Leader J. Blinn A. Freeney
b. Revenue Accounting Expense	A. Michaels
c. Commercial Business Office Expense	W. Williams and H. Chen
d. Switching Equipment Maintenance Expense	B. Murphy and M. Torrey
e. Transmission Facilities Maintenance Expense	H. Chen
f. Traffic Expense	R. Carlson and D. Hudson
g. Transmission Facilities Costs	H. Whitmore
h. Economies of Scale-Transmission Costs	I. Nasell

18. Programming Design and Programming	T. Stewart—Supervision
	K. Nocella, M. Ring, J. Newman, F. Donahue, R. Jones, R. McGill, M. Gitter, and K. Monkmeyer—Programming
19. Programming (Data)	E. Fuchs—Module Leader
	A. Juliano—Supervision
a. Point-to-Point Analysis	K. Conroy T. Angelini B. Culhane B. Jorrisch E. Rouse A. Ciccone E. Vanden Heuvel

b. Costs Analysis	M. Lessner J. Hopson T. Ebner C. McGarrigal P. Tompkins A. Golten M. Murdoch
20. Documentation	E. Smith—Module Leader J. Wangelin—Coordination and Progress Reports S. Rokos—Technical Documentation and Supervision N. Meier, C. Petuch, R. Degnan, (BTL), and R. Weis (AT&T)—Documenters J. Green—Flowcharting P. Shapiro—Illustrations

1. Introduction

Two issues with which a public utility is continually confronted are how to structure its service offerings to satisfy the changing needs of consumers and how to develop pricing policies for these offerings which will optimally serve the interests of the public, of society, and of the company itself. In the case of the Bell System, these considerations have been made complex by the multiplicity of switched network service offerings which have been created to satisfy demand, and by the various rate or price categories that have been structured in an attempt to allocate resources efficiently and thus provide a wide range of services at low costs. With the present trend toward offering even more varied services, with corresponding differentiations in price (such as the point-to-point pricing concept) indications are that this complexity will continue to grow.

This work addresses the problems inherent in studying the Bell System's price-cost relationships by describing a model that was developed as both a management and a research tool. The model was created with the intention of providing a means by which management could evaluate and analyze price-cost interactions in the network services, and at the same time providing a research mechanism or tool for the study of other model formulations and approaches, perhaps for different aggregations or functional makeups.

The reader will notice that the model which we describe and the methodology which was used rely relatively little on classical economic theory of the firm. The standard texts, such as Henderson and Quandt ([3]) present models of the firm which are primarily static models of a single-commodity firm under conditions of competition. Authors such as Chamberlin ([2]) have also given some attention to the problems of firms facing monopolistic or oligopolistic market conditions; however, these also tend to be static and single commodity models. Some more recent work has attempted to deal with the conditions imposed by some forms of regulation (Averch and Johnson, 1962 [1]). Again, however, the models depend in essential ways on a very special view of the firm.

Due partly to the lack of an adequate theoretical framework, this model does not attempt to present a general theory on which to base optimal decision-making about the various choices that are available to the System. Rather, the model is intended as a tool with which one can explore the probable outcomes of some of the choices which are available. Many other factors, both within and outside the System, must be considered before an optimal choice could be made in any given situation.

Concerning the collection, exploration, and analysis of the large mass of data which were used in this study we have kept clearly in mind the limitations on the accuracy and definitions of such data. (See Morgenstern [5]). In fact, the approach taken in the entire study, both in the handling of the data and in the formulation of the model, was much influenced by Morgenstern's works in these areas.

In summary, the purpose of this model is (to paraphrase R. W. Hamming) to provide insight, not decisions.

2. Study objectives

The primary goal of the modeling work was to build a research facility that would systematically couple costs and revenues associated with toll telephone usage, so as to provide a flexible basis for evaluating price-cost relationships in the Bell System. Such a facility would be of specific value in the determination of pricing policies, and should contribute in general to the entire decision-making process. The following capabilities had to be realized in the final model design, if these purposes were to be served:

(a) The model must be capable of providing total estimates of incremental costs and revenues[1] for MTS and WATS[2] market changes.

(b) The output should provide insight into the structuring of MTS and WATS services according to price. It should therefore give incremental estimates of demand and cost by the major rate parameters for each of these services wherever possible. Attainment of this objective is difficult, because of the nature of the economic process being studied. That is, for economic analysis the firm must be viewed as a producer of many products, the nature of each being determined by the rate parameters associated with the service offered. (These parameters are described in Section 5. pg. 15.) The major portion of the firm's plant is used jointly in the production of these varied products. This sharing creates problems in the definition of an economic theory on which one can base a modeling methodology which will be adequate for developing estimates of the elements of demand related to specific cost categories — regulatory jurisdictions (inter- and intrastate); time-of-day and rate-mileage classifications; peak and off-peak usage; and so on. Demand is also classified according to various types of service, such as direct distance-dialed and operator-assisted, and appropriate cost estimates must be developed related to these categories, which are nested within the time, distance, and regulatory classifications.

(c) In estimating incremental costs associated with the major price parameters, the model must be capable of evaluating alternative rate change proposals as they affect plant utilization, volatility of usage, and revenue characteristics, *e.g.*, peak period characteristics of the network.

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[1] "Incremental costs" are additional costs associated with providing additional units of service (required by a demand change) over and above the costs that would be incurred had the additional units not been offered. The period of time over which costs are incurred also requires definition; economists have suggested a period "long enough that all relevant costs — capital, labor, and technology — will be variable." "Incremental revenues" are defined, similarly, as the additional revenues which would accrue to the Bell System as a result of its providing incremental units of service (to meet a change in demand).

[2] Message Telecommunications Service and Wide Area Telephone Service, respectively, the two major offerings which serve toll telephone demand.

A second objective of the modeling activity was to provide a mechanism for the study of the Bell System's extremely complex price/demand/cost relationships, in more or less real (or compressed) time. The complex interactions of this economic process could then be analyzed from an internal standpoint, and valuable insight might be afforded into the relative significance of the major variables employed.

Third, the model should be structured as a research tool capable of providing insight into the validity and performance of various modeling methodologies. Other econometric models of the demand-supply or price-cost relationships can be researched within this framework as well, and their validity analyzed.

Finally, the design of the model system should provide a means for coordinating and organizing information about the demand-supply process, and in so doing should stimulate managerial attention toward areas of importance for further study. Attainment of these objectives made it desirable that the model be designed and structured in an evolutionary way, with its basic architecture forming the framework within which refinements could be developed and tested.

3. Service and pricing considerations

In the customer's view, telephone service gives one the capability of communicating not only with telephones in the local area but also with telephones in every exchange throughout the country. Even though the market is undergoing continual change, the customer feels that this capability should be provided so that his needs will at all times be met satisfactorily — meaning that the desired volume, type, and grade of communication service will always be available on an economical basis on demand, and that this service will have such characteristics as good transmission quality, acceptable mechanical performance, and a high frequency of success in completion of calls within a minimum length of time.

From the industry's standpoint, telephone service — a service rendered to the public through a physical system — has quite different values. The value of the service accrues not merely from the existence and operation of the physical facilities, but also from the development of telephone patronage through appropriate pricing policies.

Rates charged for telephone service are set on the basis of both cost and value elements, in an attempt to reconcile these two seemingly disparate views. Prices are set to reflect to some degree the relative value which customers assign (as a group) to a specific service or subclass of service. While it is recognized that demand for some kinds or elements of service will not necessarily be reduced by relatively higher prices, a telephone company must take into account the possible elasticity of demand for other services (or subclasses of a service) in setting prices. And, of course, the company must consider the cost of providing the service when setting prices.

The selection of service offerings and prices performs several functions. One function is to optimize the allocation of resources among competing uses; there are limited capital and labor resources available to the utility, and they must be used in ways which maximize their effectiveness. Another function is to attract capital through recovery of costs and realization of profits. A third is to stimulate efficiency through matching capacity to demand while attempting to minimize costs.

If the price of a service or of a subclass of service is properly set, the customer will be able to receive the service in the quantity that he needs or desires. Through appropriate pricing, social goals can also be achieved, such as subsidies to low-income groups or the like.

Costs play an important role in pricing policies, in that they not only serve as a base from which prices naturally evolve but also provide standards by which regulatory commissions can evaluate the reasonableness of rates. Public utilities are subject to regulation of rates, services, accounts, facilities, and interconnections. Such regulation was established to protect the public interest, insure adequate service, and act as a substitute for some of the forces of free competition.

This is not to say, however, that rates or prices are entirely dependent on costs or the value ascribed to a service by the customer alone. On the contrary, many other factors also

affect the determination of price levels. Corporate earnings, for example, must be stable enough to permit the utility to provide a dependable and satisfactory grade of service on a long-run basis by supporting the research and development required; in order to maintain stable earnings at levels which are both economically and legally justifiable, prices must be carefully determined and adjusted from time to time. The issue of equity also has to be considered — that is, the rates charged for services must be competitive and just, as compared with those charged for other services offered by the same company or by other companies.

In addition to all of these factors, the history associated with present rates must be considered when management evaluates service offerings and pricing policies. That is, the existing rate structure — by definition, a point of departure in rate-making — represents quasi-obligations assumed by past and present pricing policies, and must therefore be considered in any service or rate change decisions.

Rate-making, it is said, is an art and not a science. If that is so, then it is clear that tools must be developed which can refine the techniques of demand and cost analysis so as to advance the state of the art by affording maximum insight to its practitioners. This is one of the main goals which the AT&T toll network model was intended to satisfy.

4. Characteristics of the major network service offerings

Before the model can be described meaningfully, it will be necessary to define the problem — that is, to give some background information on Bell System switched services and associated supply systems. The major network services that will be of concern in describing this price-cost relationship model are:

(a) Local telephone service (as it interacts with toll service),

(b) Toll Message Telecommunications Service (MTS), partitioned by regulatory jurisdictions into interstate and intrastate usage categories,

(c) Wide Area Telephone Service (WATS), divided into outward and inward categories of service and also partitioned by regulatory jurisdictions into interstate and intrastate usage categories, and

(d) Other miscellaneous toll services, *e.g.*, Dataphone. (These comprise a small percentage of the toll usage.)

The first network service, local exchange or local telephone service, provides communication service between subscribers within a defined local area. This service is priced either on a flat rate basis (for an unlimited amount of usage within this defined local area) or on a message rate basis (permitting a certain number of calls or message units for a basic charge, and levying an additional charge per additional call or unit).

The second network service, Toll Message Telecommunications Service (MTS), also called long distance telephone service, provides two-way conversation linkages between all telephone customers in the nation. It is available 24 hours a day, seven days a week, and requires a vast network of facilities to provide this full-time capability. Calls are priced on a per-use basis, and vary with the type of usage. This service offers great flexibility in the adaptation of facilities to serve varied traffic loads. It also offers to the customer assisted calls of various types, and direct distance dialing (DDD), all of which make it an efficient and economical service offering.

The third type of service of concern to this analysis is Wide Area Telephone Service (WATS). This is a form of long distance service developed principally for users with large-volume needs — business customers — whose calling areas may cover wide geographic zones. The WATS customer may select from a range of possibilities the area to which he wishes to have service, and he may elect to have that service provided on either a measured-time monthly basis or a full-time basis. Within the area which he selects, he has access to all customer telephones.

There are nested categories of WATS service from which the customer must choose to satisfy his particular need. He may prefer inward or outward service, with measured or full-time access. Briefly, these offer the following capabilities:

(a) Outward WATS — Service from the customer location to all telephones within the area specified in the customer contract.

(b) Inward WATS — Service from all telephones within the area under contract to the customer's location.

(c) Measured WATS — Service providing limited usage within one of the above categories, measured in time units and billed at a fixed charge for a base usage plus an overtime charge for additional time.

(d) Full-time WATS — Service providing access to or from all telephones within the area specified in the contract on an unlimited basis for a flat monthly rate.

Outward WATS was offered for the first time in 1961, while the inward service did not appear until 1970.

Another significant partitioning of services and prices results from the regulatory jurisdictional structure. The intrastate service categories — local exchange and the local portions of MTS and WATS services — come under the jurisdiction of state regulatory commissions, while the interstate services are regulated by the Federal Communications Commission. However, in the interests of allocating resources as efficiently as possible, the telephone companies use a major portion of their telephone plant for both intra- and interstate services in common. It is necessary to segregate the amounts of telephone plant (and the operating expenses associated with those amounts) so that for analytical, accounting, or regulatory purposes we can specify what revenues and costs are associated with intrastate usage and what with interstate. But determining the appropriate separations methods is difficult, and the means chosen are dependent upon the objective — whether it be to derive data for analysis or to meet regulatory needs. Procedures for allocating costs and revenues for regulatory separations between state and interstate jurisdictions have been outlined by the National Association of Regulatory Utility Commissioners (NARUC); these are used to develop overall rate bases for state and Federal regulatory purposes and for allocating revenues among the Bell System Associated Companies and the Independent Telephone Companies. However, they are not adequate procedures for the study of demand for individual services or of price-cost relationships. Thus, a separate module was developed to study the effects of separations procedures and results.

Another important aspect of the price-demand relationship is related to the price structure. There are four major parameters used in setting the price of calls under the MTS service offering:

(1) Service class:

  (a) Customer-dialed. This is the least expensive, most customer efficient, and most economical in terms of use of telephone resources (labor and facilities).

  (b) Operator-assisted station-to-station. This requires the assistance of an operator and some specialized equipment to perform an added service (such as assisting in dialing, computing the charges for a call, or the like).

  (c) Operator-assisted person-to-person. This is the most expensive service class, and requires the greatest amount of personal operator intervention. The operator assists by locating the specific called party. (No charge is made for incomplete attempts.)

(d) No charge. This class includes directory assistance, charge and time information, and other assistance.

(2) Rate periods:

The concept of charging different prices for different periods of day (and days of week) is used to provide toll service to customers at reduced prices for efficient utilization of the network. One way of obtaining greater efficiency in the network is to stimulate usage of facilities during off-peak periods, whenever possible shifting demand from peak periods. This may be accomplished by lowering rates during the non-peak periods, making it more attractive to the customer to place calls during those times of day. This shift of usage from the peak to the non peak period makes use of the spare capacity which exists in the network, and since plant is used in common by other services and classes of service, the economic benefits of the shift are realized in all areas of the long-distance telephone network.

(3) Timing periods:

The period of time that elapses during a telephone call is broken down into segments. These segments can be related to certain required equipment functions. There is a start-up or connection charge ("initial period rate"), and an overtime charge. The initial period rate for a call is designed to recover the set-up costs, and the overtime charge is designed to recover the marginal costs associated with the holding time of the call. One objective of using timing periods is to attempt to assign to the customer the costs for the amount of service which he actually uses. Another is to provide the customer with greater flexibility in satisfying his communications needs.

(4) Distance or mileage differential:

The unit cost of circuits varies with distance. Therefore, a mileage differential is applied in rates for long-distance calls. With more recent technology. such as various microwave developments, the costs incurred at the terminations of the long distance circuits represent an even greater proportion of the total cost of transmitting a call than before. Costs of circuits vary by types of circuit facilities, which depend upon geography, length of haul, and traffic density. All of these circuit costs are factors that must be considered when rates are being set.

Three major parameters are used to set prices for WATS service:

(1) The first is the category of service — outward or inward calling — which was described previously.

(2) The second is class of service — full time or measured time — as described previously.

(3) The third criterion is the size of the service area. WATS zones have been defined to give the customer a selection of calling areas that will best meet his needs. There are six WATS zones; Area 1 includes the nearby states, and the next five zones radiate out to cover progressively larger areas until the entire United States (excluding Alaska) is covered (by Area 6).

Composition of toll telephone traffic. The composition of the total traffic (including local and toll) that is offered to the various components of telephone plant is one critical aspect of the problem, and therefore must be recognized and handled by models designed to investigate price-cost relationships. Table 1 gives the approximate message levels for various categories of traffic during an average business day (ABD) for the major switched network services. It also gives the composition of the typical load, in characteristics which vary significantly within the message volumes recorded. All of the traffic elements interact dynamically, and it is this dynamic structure which must be taken into account in the modeling of price-cost relationships.

TABLE I

COMPOSITION OF SWITCHED NETWORK SERVICE TRAFFIC

CATEGORIES OF SERVICE	ABD MESSAGES*
LOCAL	330
LD INTERSTATE	8
LD INTRASTATE	12
WATS INTERSTATE	1
WATS INTRASTATE	

COMPOSITION-OF-LOAD VARIABLES

CLASS OF CALL: DDD, STATION-TO-STATION, OPERATOR-ASSISTED
CLASS OF SERVICE: WATS INWARD, WATS OUTWARD, MTS BUSINESS, MTS RESIDENTIAL, MTS PUBLIC (COIN TELEPHONE)
RATE JURISDICTION: INTERSTATE, INTRASTATE
HOUR OF DAY
DAY OF WEEK
SEASON
GEOGRAPHIC LOCATION
DISTANCE CALLED (LENGTH OF HAUL)

* Message units are millions.

Figures 1 through 4 illustrate the composition of the toll traffic for an average business day (ABD) during a study period in March, 1968; this traffic comprises three major service classifications — WATS, MTS and other interstate, and MTS intrastate. Figure 1 illustrates that (1) load varies by time of day; (2) the contributions of the various toll services to the total load vary over the hours of the day; and (3) these patterns of load vary among geographical regions. (For all four figures, the total Bell System distribution of "base" load is shown, together with the loads at three of the ten regional switching centers[3] located within the United States.) Figure 2 illustrates that (1) there is variation in load over the rate mileage bands developed for the model; (2) the contributions of the various toll services vary over hours of the day; and (3) these patterns differ between geographical regions. These variations pinpoint areas within the price structuring problem that are of vital interest in modeling telephone demand. Another important characteristic of telephone traffic is related

[3] The hierarchy of switching centers is discussed in a later portion of this section. (See Section: Structure of the Network p. 17)

to the variations that exist internal to the price structure of each of the services.

FIGURE I

HOURLY DISTRIBUTION OF CCS LOAD

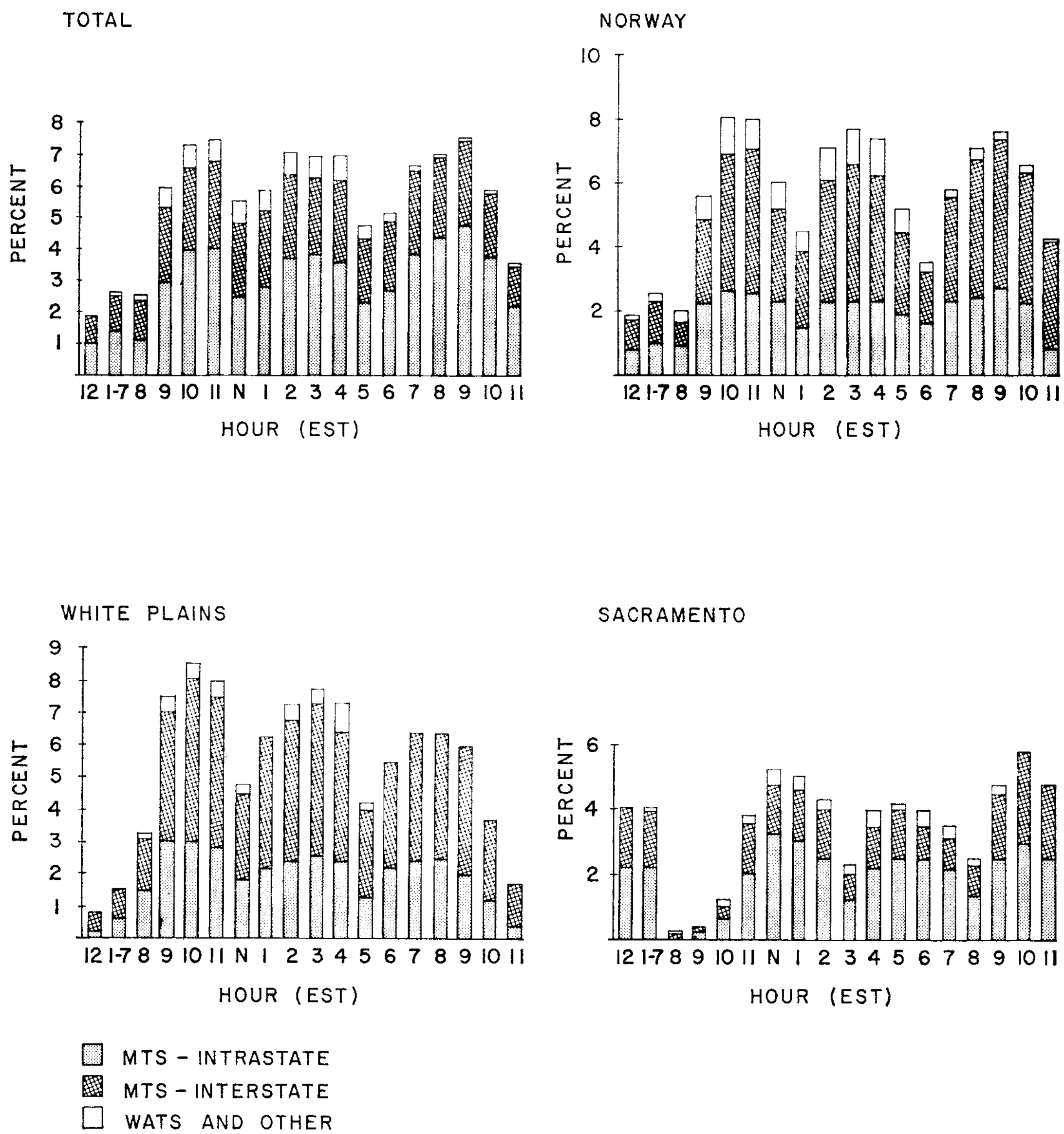


FIGURE 2

COMPOSITION OF BASE LOAD BY MAJOR CLASSIFICATION

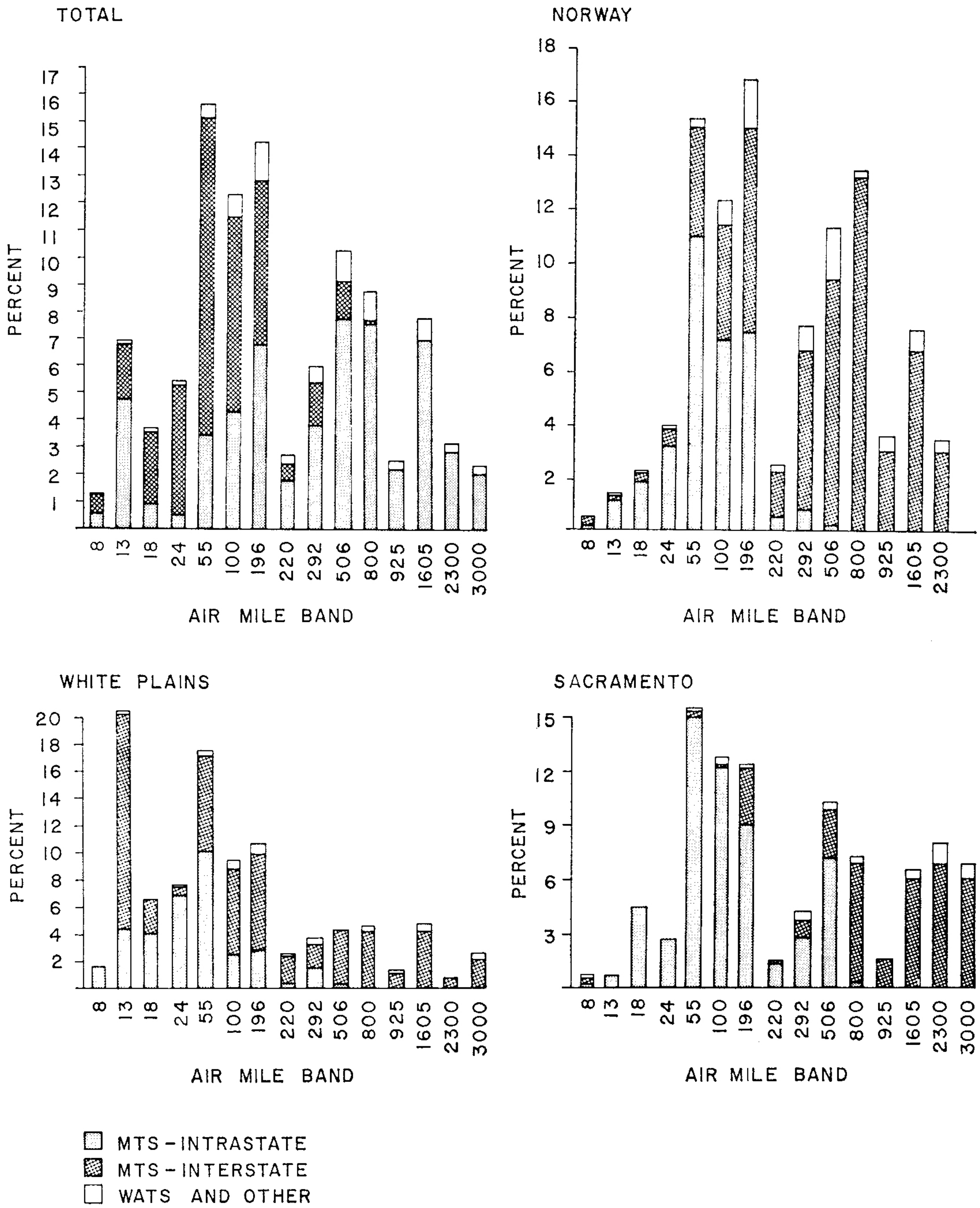
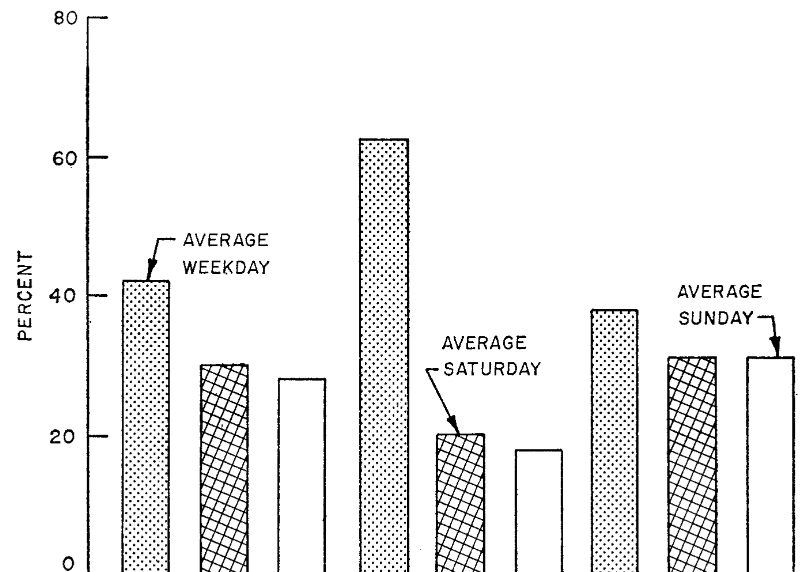


Figure 3 illustrates how interstate MTS demand varies by classes of call, by day of the week (average business day, Saturday, or Sunday). These variations differ, depending on whether one views traffic in terms of local or of common time. The latter distributions are particularly important, because price is a function of local time, while costs are based on common time because the system is designed to serve all demands at a given point in time simultaneously. This feature will be explained in greater detail later in this section.

FIGURE 3

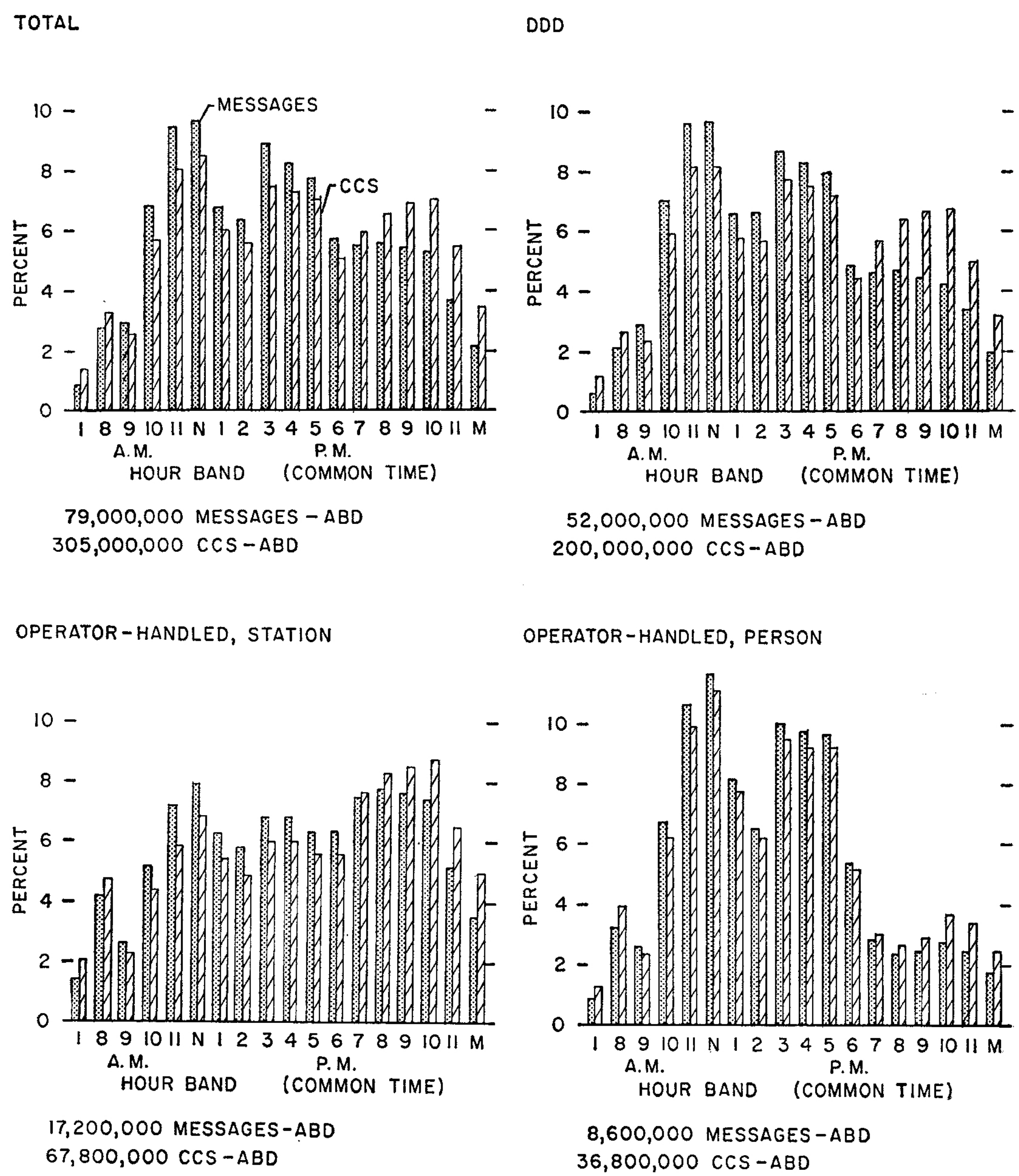
DISTRIBUTION OF BASE INTERSTATE MESSAGES



TYPE OF MESSAGE	DDD			PERSON – PERSON			STATION – STATION		
TOTALS BY DAY & TYPE OF CALL (THOUSANDS)	5,460	3,900	3,640	899	290	261	1,960	1,470	1,470
TOTALS BY TYPE OF CALL (THOUSANDS)	13,000			1,450			4,900		
TOTAL DEMAND, BASE (THOUSANDS)	19,350								

Figure 4 illustrates that, within each class of call for the interstate MTS ABD load, variation exists by time of day for both messages and holding time (expressed in CCS, hundred call seconds). These figures (1 through 4) are included to illustrate certain demand characteristics which, once recognized, required use of a disaggregate approach for the study of demand-cost relationships.

FIGURE 4.
HOURLY DISTRIBUTION OF MESSAGES AND CCS LOADS



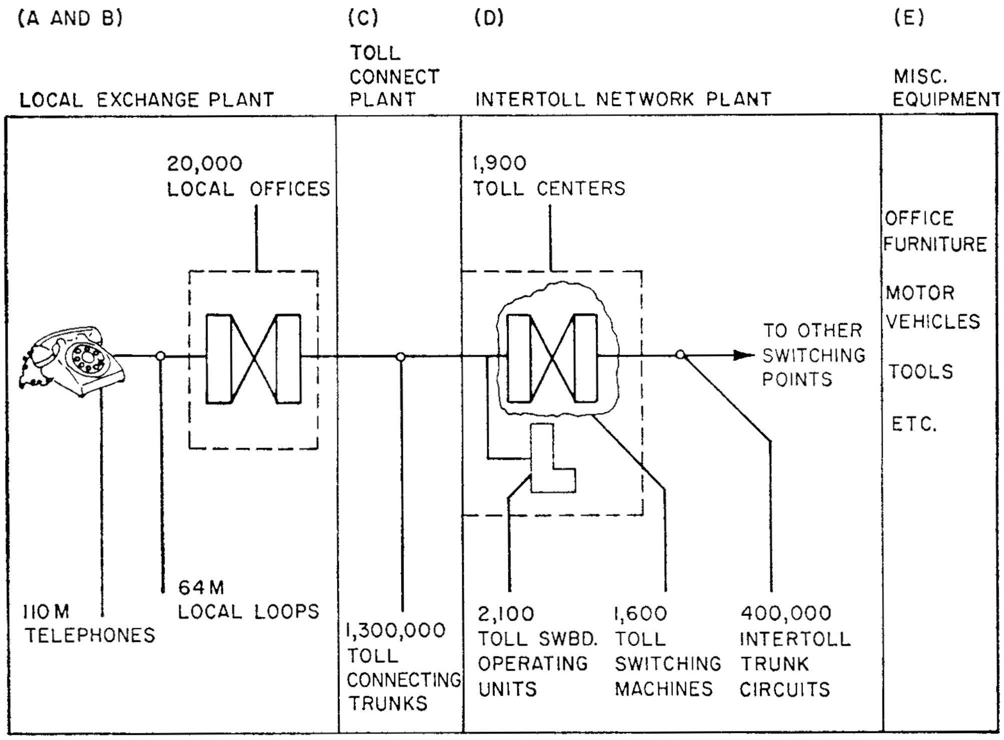
5. Characteristics of the network supply system

The network over which telephone calls are carried comprises millions of miles of wire, cable, and microwave relay trunks, a large number of switching machines, and numerous network management controls. A basic characteristic of this network is that it is shared among Bell System service offerings, and that it must therefore deal with many types of demand simultaneously.

In order to visualize this system, it may help to think of the analogy of a widely distributed set of hierarchically organized switches. These may be considered to be under the control of a large real-time computer. Any subscriber, at any time, may contact the computer by picking up the handset of his telephone. On the subscriber's instruction, the computer can cause the appropriate switches to be set so that the subscriber is electronically connected with any other subscriber telephone anywhere in the country. Of course, the computer also controls the detection of busy conditions, selection of alternate paths where possible, resetting of switches on termination of calls, and detailed recording of each subscriber's usage of the network. This system operates in real-time mode, essentially continuously 24 hours per day, and provides a very low probability of a "network-busy" condition throughout the day for any possible connection request from any subscriber. The actual equipment in this network has been designed and operated over a period of many years with the objective, at each point in time, of meeting detailed specific standards of minimum service quality. Within the quality-of-service constraints, the equipment and service practices have been designed to provide minimum-cost operation. The interpretation of cost is a long-term one, to reflect the fact that the telephone system is an integral part of the effective handling of information within our society.

The telephone plant which makes up this "distributed network switching system" can be partitioned as shown in Figure 5, to form an outline of the service as it has been viewed for our modeling purposes. The figure illustrates how the major categories of facilities are interconnected, and gives the approximate number of units existing within the Bell System in each of these categories.

FIGURE 5.
BELL SYSTEM NETWORK STATISTICS



From this introduction, an impression can be gained of the size, in numbers, types of service, types of equipment, and — of particular importance — range of dates of manufacture, of the switched network telephone service. The specific categories of plant are:

(1) *Local telephone (subscriber) plant*

Local plant consists of station apparatus (the equipment installed at the customer's residence or place of business, or in a telephone booth) and the transmission equipment which connects the station apparatus to the exchange switching complex (the local or customer central office).

(2) *Local exchange plant*

This category consists of the trunks which connect local central offices within an exchange area to other local central offices, or to offices used as intermediate switching points (tandem offices) for traffic between the local central offices and the toll offices.

(3) *Toll connecting plant*

The toll connecting network is made up of all the circuits connecting the exchange switching equipment to the intertoll network. These trunks carry only toll traffic (local traffic being handled within the category (2) plant), and ordinarily only provide connection between local offices (termed Class 5 in the Bell System) and toll offices (Classes 1 through 4).

(4) *Intertoll network plant*

Four sub-categories of equipment make up the intertoll network. These are:

(a) Toll switching machines, which route intertoll traffic from its originating points to its terminating points;

(b) Intertoll circuit groups, which are the trunks connecting toll switching offices and are used primarily for toll traffic;

(c) Traffic operator equipment, such as switchboards, through which operators assist in serving certain types of demand requiring their intervention (operator-assisted station or person to-person calls, directory assistance, and other miscellaneous operator-assisted calls); and

(d) Special switching equipment, such as the automatic message accounting (AMA) recording equipment, which creates a billing record of each direct-distance-dialed call (noting the time of day, length of conversation, and so on, for billing purposes).

(5) *Miscellaneous equipment*

Into this category are gathered all of the acquisitions and appurtenances that are external to the physical transmission of telephone calls but are nonetheless essential to the support of the equipment, such as furnishings for offices, motor vehicles used to transport workmen and equipment, tools and supplies required for maintenance and installation, and so on.

Structure of the network. An understanding of the structure of the network is fundamental to the formulation of supply-cost models of the toll network services. Within this structure, customer and usage characteristics have led to the development of detailed design criteria. The subscribers who may desire toll telephone service are distributed in varying densities and configurations over any given area. Each subscriber represents a point which must be connected, on demand, by a two-way communication channel or circuit to any one of the several million other similar points in the system. Now, if every one of these points were to have a direct line to every other one, it would be necessary to install $n(n-1)/2$ lines, where n is the number of points. This is obviously not practical for more than a very few points, and it was therefore necessary to develop an acceptable alternative in designing the network.

By applying switching at certain intervening locations between the points, it is possible to serve all points by permitting them to share a hierarchically structured set of facilities. Each subscriber line is then terminated at the nearest of an array of central offices (Class 5 end offices), each of which covers a defined area. The number and location of these offices are determined by engineering studies aimed at achieving the optimum balance between subscriber line costs and the costs of additional switching and trunking.

A system of switching has also been devised for the trunking facilities between switching centers. Generally, of any group of subscribers, only a small percentage will require entry to the network at a given time. Therefore, through switching, traffic can be combined to form a number of large, efficient circuit groups interconnecting various toll centers in the network, instead of the small, costly, and inefficient groups which would result were these connections provided on an individual point-to-point basis. The fundamental objective of this switching system is to attain an economic balance between the costs of direct trunking and tandem, or alternate, trunking.

The toll switching network is hierarchical, partitioned into 12 major geographical areas or regions of which ten are in the United States and two in Canada. Figure 6 shows the twelve regions, with the cities which are known as regional centers. (Note that the distributions shown in Figures 1 through 4 applied to traffic through 3 of these centers — White Plains, Sacramento, and Norway.) Within each of the twelve regions is a Class 1 toll switching office. These form the pinnacle of the toll switching hierarchy.

Figure 7 shows the pyramidal arrangement of toll centers in the network, and the so-called "homing" pattern by which they are interconnected. Each office is classified according to the type of switching function performed, its relationship to other offices, and its transmission requirements. Besides the Class 1 regional offices, there are Class 2, or sectional, offices; Class 3, or primary area, offices; and Class 4, or toll center, offices. Finally, the Class 5 offices, known as end or local offices, are connected to the subscriber lines and are the means by which the customers' requests for service first enter the network. The higher the number of the class, the more localized the area served and the greater the number of offices in the class.

FIGURE 6
UNITED STATES AND CANADA REGIONAL AREAS

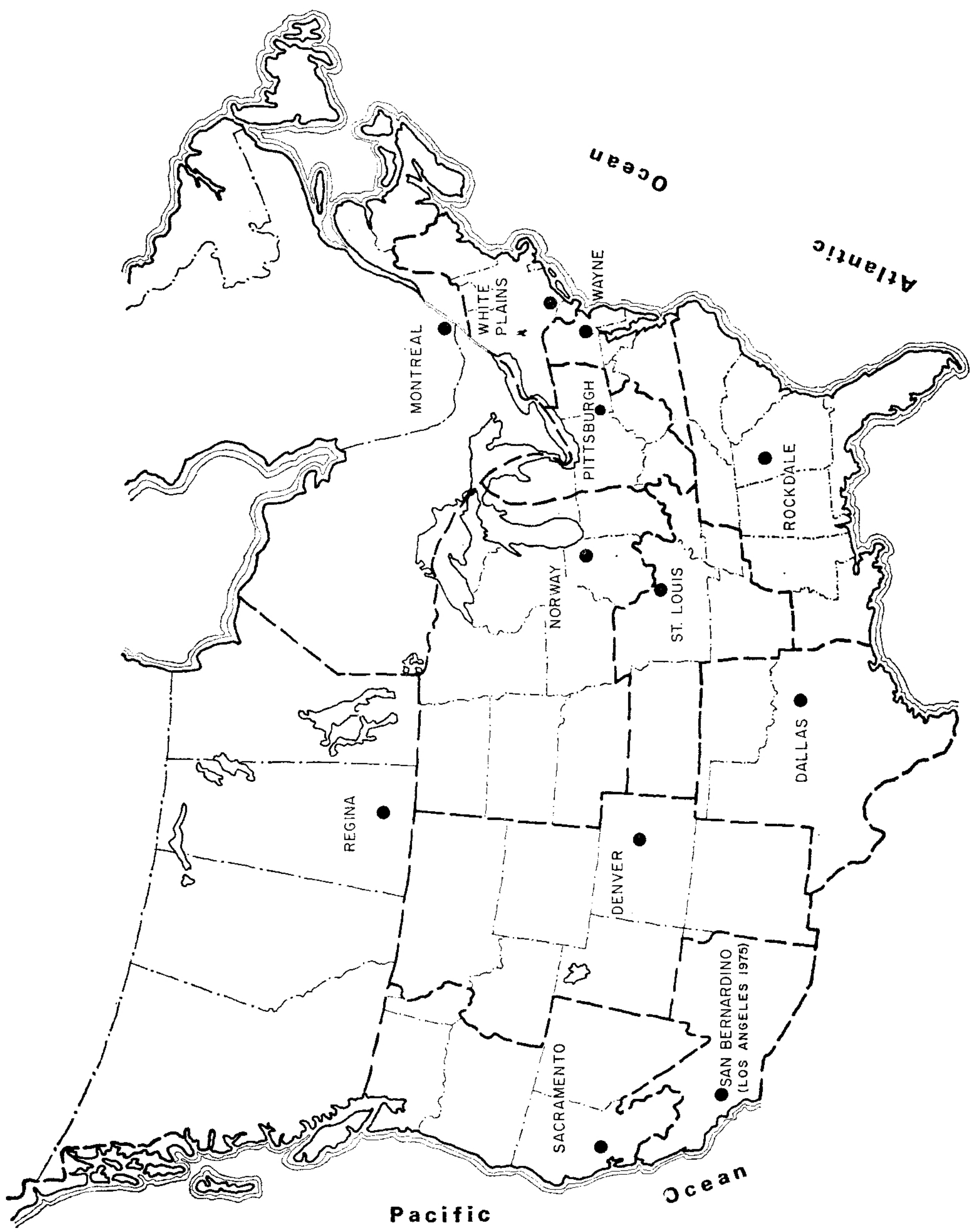
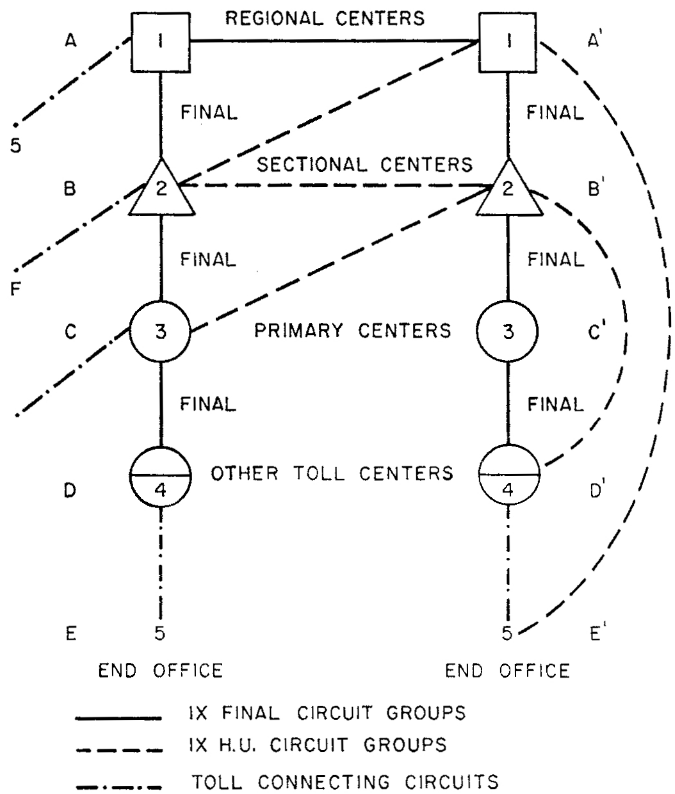


FIGURE 7.

HOMING ARRANGEMENT OF TOLL CENTERS



NOTES:

CLASS 5 OFFICES CAN BE CONNECTED DIRECTLY TO CLASS I, AND CLASS 4 TO CLASS 2. A CALL FROM END OFFICE E TO END OFFICE E' WOULD (FOLLOWING PRINCIPLES OF ALTERNATE ROUTING) TRAVEL UP LADDER TO PRIMARY CENTER C, WHERE IT WOULD BE OFFERED TO FIRST-CHOICE HIGH USAGE ROUTE CONNECTING PRIMARY CENTER C WITH SECTIONAL CENTER B'. IT WOULD THEN BE CARRIED DOWN LADDER, B' TO C' TO D' TO E'.

The switching plant is located within these centers and is interconnected by circuit groups. There are three fundamental aspects of switching plant which may vary between centers:

(1) The type of apparatus provided,

(2) The interconnecting network within the switching office, which performs the connection of incoming calls on subscriber lines or intertoll trunks to the desired subscriber lines or trunks, and

(3) The control apparatus which causes the switching machine to perform by selecting the proper path between lines or trunks.

In the design of a switching center, each of these aspects influences the others, and the total configuration which results is the product of their interaction. There are basically two types of switching equipment in use in the Bell System, the direct-dial-controlled type and the common-control type. In the first, a call proceeds through the system from switch to switch in direct response to signals generated by a subscriber's or an operator's telephone dial. Each switch in the train, when it is "seized" after the dial signal has been received, hunts for an available idle path to the next switch. Step-by-step equipment (so-called because the switches "step" from position to position sequentially as the number is dialed) is of this type. In common-control switching, control equipment which operates the switches and sets up connections indicated by the received signals from subscriber or operator dials is shared by the whole switching office. This equipment first identifies the incoming and outgoing terminals, then selects an idle path through the system and operates the switches to establish the connection. This function includes selection of alternate routing when busy conditions are encountered. The Bell System's No. 5 Crossbar and No. 4 Toll Crossbar, Crossbar Tandem, and Electronic Switching equipment are all of this type. These are costlier machines for small offices than are the direct-dial-controlled type, but they have the advantage of offering greater flexibility.

The type of equipment selected for a particular location depends upon many factors, among which are office size (number of lines or trunks served), rate of growth within the area served, proximity to other offices, kinds of service required, present technologies, and individual management preference. The growth rate determines the economic engineering interval, which is the period between additions of new equipment. The main equipment selection problem is to maintain a balance of capacity with load within the office such that each of the various switching elements (control, handling, and terminating) is provided at a satisfactory level within the engineering interval, which will differ from element to element (as will margin of spare capacity).

TABLE 2

TOLL SWITCHING OFFICE EQUIPMENT (BELL SYSTEM)

OFFICE CLASS	EQUIPMENT TYPE						
	4A	4M	XBT	5XB	SXS	INDP	TOTAL
1	10	0	0	0	0	0	10
2	39	6	8	0	0	1	54
3	18	0	117	54	9	16	214
4	0	0	76	341	559	563	1,539
TOTAL	67	6	201	395	568	580	1,817

Table 2 gives the various types of switching equipment by class of office (within the switching hierarchy) presently in use within Bell System and Independent telephone companies. The analysis of Bell System price-cost relationships was structured to investigate the ways in which changes in demand affect switching costs, because the relative economy of installing one type of switching equipment as compared with another varies as a function of demand.

Interconnecting these toll switching offices is a complex trunking network. The trunks connecting the end offices to the toll switching offices are called toll connecting trunks. Each office is connected by an interchange group of trunks to its “home” (next higher ranking) office, and that group is called its “final” group; in Figure 7, final groups are represented by solid lines. Within regions, calls are carried on circuits up and down the ladder shown on either side of the diagram. Between regions, two ladders are involved, and each must have its regional center connected by a final trunk group to every other regional center. Between offices of other classes, placed wherever the volume of traffic warrants installation of a direct connecting facility, are the so-called “high-usage” trunks. These are shown by the dashed lines in Figure 7.

The provision of circuits in the intertoll network is governed by a criterion which is known as the blocking probability, or factor: the probability of encountering a “no-available-circuit” condition during the average busy season-busy hour must be less than a specified percentage for any call entering the network. The high-usage, or direct, trunk groups are engineered so that a portion of the traffic seen at the busy hour on any given route will overflow to the next alternate high usage group, if there is one, and eventually to the final group.

The principle of alternate routing now needs to be reviewed. This principle is extremely important to the economic design of communications switching networks (and thus to a model of a communications network), and is based on the fact illustrated earlier in Figures 1 through 4, that the volume of calls originating at points throughout the network varies significantly from season to season, day to day, and hour to hour. Given the load behavior, it would not be economical to design the network to serve all traffic between points over a single unique path for each pair of points. Instead, to make more intensive use of facilities and to reduce costs, alternate routing is employed. Calls are routed over tandem routes if

they are blocked from entering the first-choice routes. Tandem trunk groups are usually large and therefore relatively efficient, and the overflow traffic from direct groups tends to come in random spurts which fill in the "valleys" — the off-peak loads — in the flow of tandem traffic. Relatively fewer trunks have to be added to the tandem group to handle the overflow, compared with those which would be required on the direct group, and this results in a net saving in trunk requirements. However, the tandem group is longer and has a switching point in it, so that it is necessary to consider the relative costs of the two routes and the call-carrying efficiency of each when determining the economic or optimum balance of traffic.

The proportion of traffic that should be permitted to overflow is determined by considering the cost of adding capacity to the high usage group rather than diverting a portion of the load. The capacity of the high-usage group which results from these cost considerations is known as the economic CCS (hundred call seconds) of the group. Economic CCS load for a given high-usage group can be found from the equation

(*) marginal cost per call_{AR} = marginal cost per call_{DR},

where AR is the alternate route and DR the direct. As the number of circuits in the direct route is increased, the marginal cost of a DR call is increased, as a consequence of the probability distribution characterizing traffic flow, while the AR marginal cost remains fairly constant. Under appropriate conditions, the economically optimal number of circuits is the solution to (*). This in tum yields the appropriate number of trunks for the direct group.

Summarizing, the final groups in the intertoll network are sized on a service basis to provide a grade of service in which the blocking factor is determined to meet a specified criterion. The high-usage (HU) trunk groups in the network are sized on an economic basis, determined by the marginal cost per call of adding a trunk to the direct route rather than allowing the call to overflow to the alternate.

The provision of alternate routing significantly increases the efficiency of the network as a whole, under certain heavy load conditions. One aspect of the alternate routing principle that is significant to a communications supply model is that further network cost savings can be realized when the characteristic non-coincidence of busy hour loads observable in the various traffic parcels sharing a common alternate route is taken into account.

Figure 8 is a schematic representation of a toll center which has associated with it two direct routes and a final group to which both direct routes overflow. Associated with each group is a plot of its average offered load as a function of time of day. Note that Group 1 has a busy hour during the morning, Group 2 during the afternoon, and Group 3 during the evening. If each trunk were sized on its busy hour load, the trunk requirements would correspond to the peaks. However, the peaks in the direct routes occur during periods when the alternate route has idle capacity. It therefore appears that if the number of circuits in the direct groups were reduced below the peak load requirements, traffic would overflow to the alternate route and would cause no increase in the final group requirement, as shown in Figure 9.

FIGURE 8.

NETWORK CONTAINING ONE COMMON FINAL AND TWO HU GROUPS

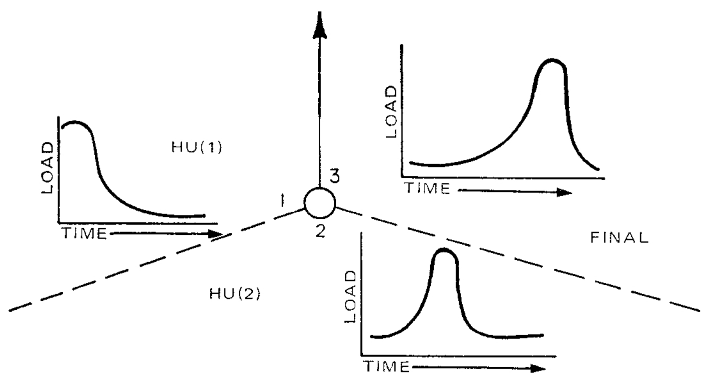
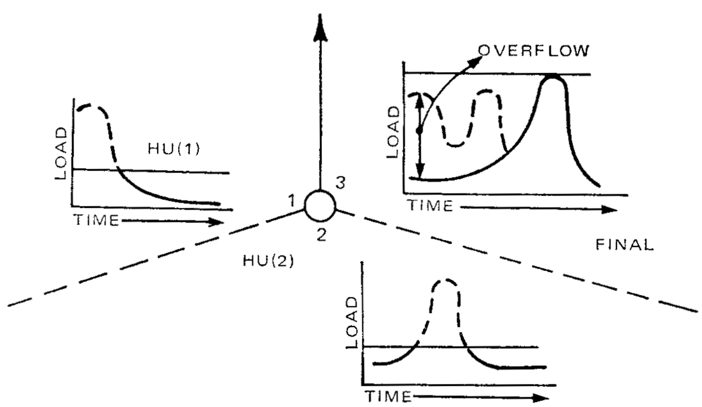


FIGURE 9.

REALLOCATION OF NON-COINCIDENT LOAD

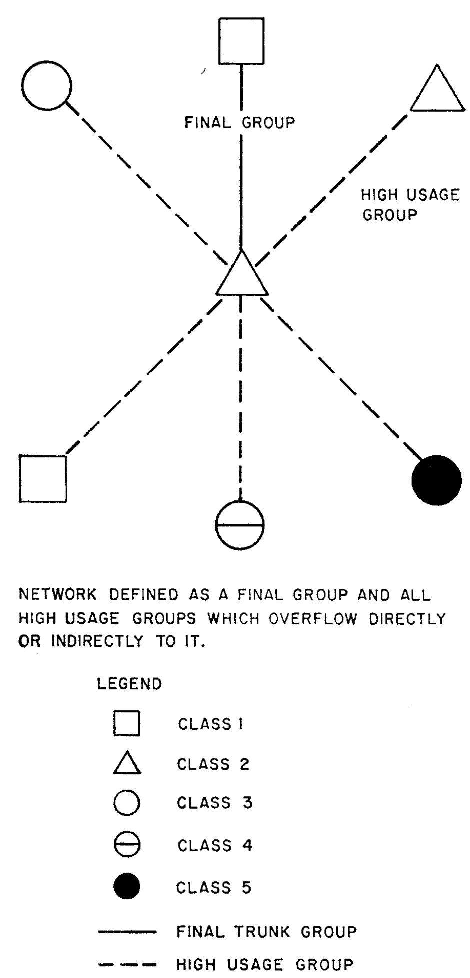


The method by which this reallocation of non-coincident load is accomplished is based on viewing each final group and all direct routes which overflow to it as a "network." The alternate routing structure of *HU* and final trunks results in a large number of networks, comprising about 1500 intra-regional and 45 regional networks within the ten United States regions. Figure 10 is a simplified illustration of this definition, which was found to be adequate for the model. Final groups connect toll centers to higher ranking toll centers which are their home offices, from which no traffic may overflow. Therefore, in this simplified network structure, every final group is in only one network and every high usage group is in at least two (one at each end of the group) and possibly three (if the circuit group is inter-regional). Each high usage group will have two or more network busy hour loads, and the largest of the loads is the one used in sizing the group.

The loads on all of the routes in a network are summed, and the resulting distribution of traffic over a twenty-four hour period is examined to determine the network busy hour (NBH). Each circuit group in the network is then sized according to its expected carried load during the NBH, and the difference in load on each route between the circuit group busy hour and the NBH is absorbed by excess capacity on the other groups in the network.

FIGURE 10.

SIMPLIFIED NETWORK STRUCTURE



Economies. We have stated that it is necessary to recognize fluctuations in load when engineering the telecommunications network. Two types of economy can be realized when these fluctuations are duly considered: size-of-scale economies, evidenced in the long-haul transmission facilities, and network economies, which arise from the use of alternate routing to satisfy point-to-point communication needs rather than direct facilities between all point pairs. These two economic concepts will now be discussed as they relate to network engineering and thus to the relationship of price to cost.

A. Economies of scale

To provide a simple description of the economies of scale concept applied to circuit engineering, average cost can be expressed as a function of circuit group size thus:

$$C(X) = KX^{-a},$$

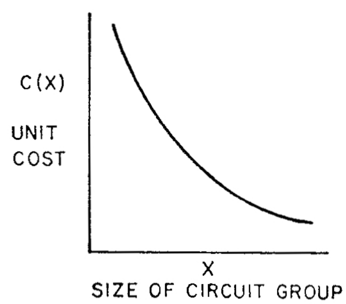
where

$$\begin{aligned} C(X) &= \text{ average cost per unit of output at level } X, \\ K &= \text{fixed average cost per unit, and} \\ a &= \text{the economies of scale parameter, } 0 < a < 1. \end{aligned}$$

Figure 11 shows this function in graph form.

FIGURE 11.

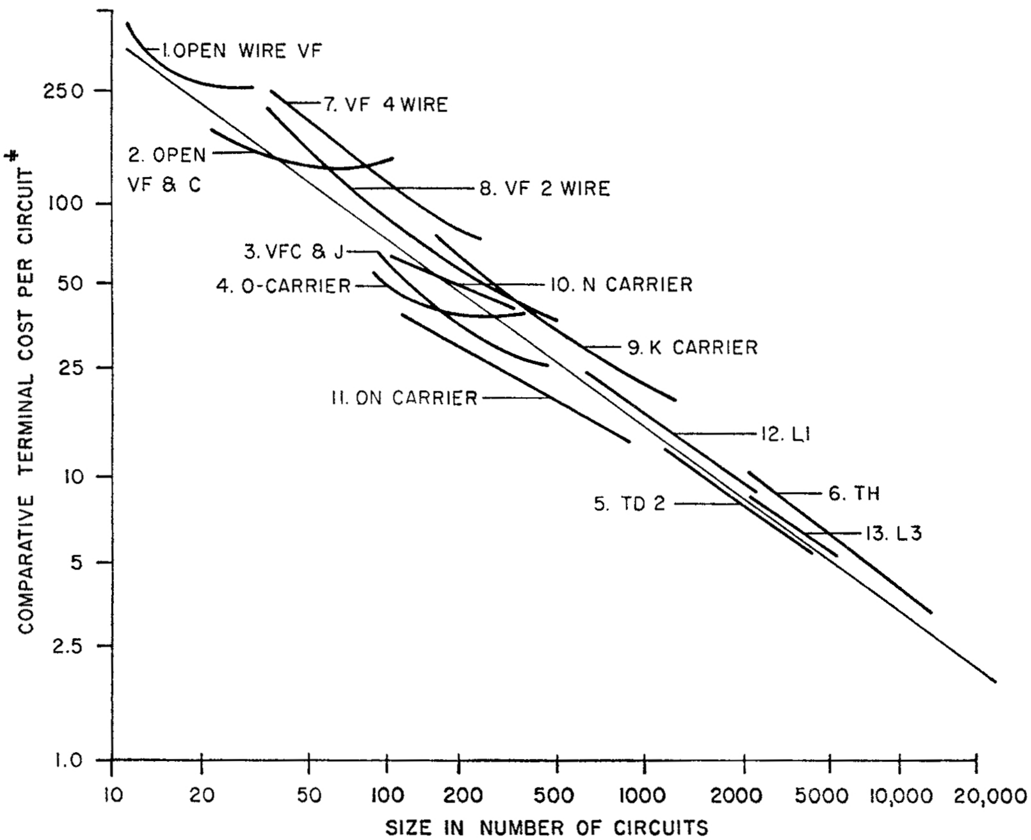
CIRCUIT GROUP SIZE VS. COST



If $C(X)$ is the average cost per circuit mile for a transmission system of size X(in circuits), the equation and the graph describe the economy of scale in long-haul transmission illustrated in Figure 12. The transmission network in which this economy can be realized is made up of a large number of different systems, each having a certain capacity. Reductions in cost have been achieved through the introduction of new technology, but often the modification of existing systems which such combinations of equipment necessitate is so expensive that the savings are greatly reduced. Table 3 gives examples of some of the major transmission systems now available in Bell System networks.

FIGURE 12

COMPARATIVE TERMINAL COSTS* PER CIRCUIT



OPEN WIRE	CABLE
1	7
2	8
3	9
4	10
	11
MICRO RELAY	COAXIAL
5	12
6	13

*REFERENCE: DIXON CLAPP CURVE -- L.R. TOWER, JR., "A NEW VERSION OF THE CIRCUIT COST VERSUS DENSITY TREND LINE", BTL TECHNICAL MEMORANDUM, OCTOBER 7, 1969

‡ TERM. COSTS NOT INCLUDED

TABLE 3

ROUTE CIRCUIT CAPACITY AND ROUTE MILEAGE

	SYSTEM*	100% FILL CIRCUIT CAPACITY	YEAR AVAILABLE
1	TD-2	3,000	1950
2	TD-21	6,000	1960
3	TD-2B	12,000	1967
4	TD-3	12,000	1967
5	TH-3	10,000	1971
6	SH RADIO	3,000	1960
7	L3-6T	3,600	1960
8	L3-8T	5,400	1960
9	L3-12T	99,000	1962
10	L3-20T	16,200	1964
11	L4-12T	18,000	1967
12	L4-20T	32,400	1967
13	L5-22T	90,000	1973
14	WG	233,855	1978
15	TD2B/TH1	12,000	1967
16	TH1/TD2B	10,800	1961
17	TD3/TH3	12,000	1971
18	TH3/TD2B	10,800	1971
19	TH3/TD3	10,800	1971
20	L3/L5-12T	36,000	1973
21	L3/L5-20T	64,800	1973
22	L4/L5-12T	27,000	1973
23	L4/L5-20T	48,600	1973
24	TH1	10,800	1971
25	L3-16T	10,800	1962
26	L4-8T	10,800	1967
27	OTHER	1,200	1950

* THESE SYSTEM NAMES ARE USED BY THE BELL SYSTEM ALONE. TD AND TH ARE MICROWAVE RADIO RELAY SYSTEMS, WHILE L DENOTES A COAXIAL CABLE SYSTEM WITH THE CORRESPONDING NUMBER OF TUBES (T). WG IS A WAVE GUIDE SYSTEM.

B. Network economies

As we have stated, the reduction in cost which can be realized through the use of alternate (indirect) routing in a network is a network economy. The fact that such economies do exist can be illustrated by simply showing that the average cost per circuit mile in the integrated network case (where alternate routing is provided) is lower than the cost in the fully connected case (where direct routes between all points are installed).

Let the average cost per circuit be represented as

$$C(X) = \big(K \sum_{all\ l} X_l^{-\alpha}\big)/m, \quad X_l > 0, \quad 0 < \alpha < 1,$$

where

$$\begin{aligned} C(X) &= \text{ average cost per circuit mile for } X \text{ circuits,} \\ K &= \text{ fixed average cost per circuit,} \\ \alpha &= \text{economies of scale parameter,} \\ m &= \text{total circuit miles, and} \\ l &= \text{links.} \end{aligned}$$

Comparing the two cases, one an integrated network where alternate routing is provided, the other a network containing direct routing between point pairs, the difference will represent the reduction in costs resulting from the network effect, or

$$\text{Network economies} = C'(X') - C(X),$$

where

$$C'(X') = \text{average cost per circuit in the integrated network.}$$

Analyses of specific situations, using this model, show very substantial network economies.

The principles which have been described were considered critical to the formulation of a supply model to study price-cost relationships, and are given in more detail in the Traffic Facilities Practices relative to intertoll trunks, published by AT&T as the recommended engineering procedure for the Bell System. The model was structured so as to reflect these major engineering principles, in order to determine price demand effects on the approximately 15 thousand intertoll links connecting some 1900 toll centers within the Bell System's wholly and partly owned subsidiary companies.

Traffic-network supply relationships. The characteristics of the switched network plant and the traffic offered to various components of that plant vary, often in an extreme fashion. These characteristics are fundamental to the design and structure of any analysis undertaken in order to investigate price-cost modeling methodologies, and therefore have to be understood in some detail. The diversity of the network plant results in variations in

(a) Density of circuits,

(b) Circuit length,

(c) Physical environment,

(d) Quantities needed (*i.e.*, economic size of the facility),

(e) Number of other services using the same facilities, and

(f) Life of the facilities.

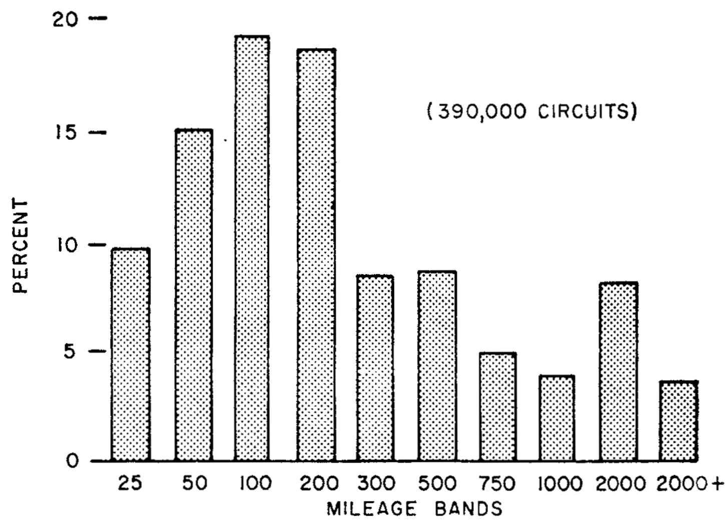
Figures 13 and 14 provide additional insight into the characteristics of telephone plant. Figure 13 shows how the interexchange (IX) network circuits vary by length of haul. It also shows the composition of the basic load offered to the various lengths of circuit by major service classification, so that the distributions shown are functions of both demand and geography. Figure 14 shows how the IX circuit groups vary in size (*i.e.*, density), for all circuits and for three aggregate lengths of haul — 100, 1000, and 2000 miles. It can be seen that there is a wide range of circuit group sizes within each mileage band. This variation is a function of demand by time of day, geography, and non coincidence of load, all of which are utilized in the engineering of the network to achieve economies of scale and network economies.

Since equipment is engineered to satisfy peak capacity demand, the traffic characteristics associated with the capacity requirements are critical to any study of price-cost relationships. The next set of figures gives some insight into the demand busy-hour characteristics of the various components of plant when the network has been designed to realize scale and network economies. Figure 15 illustrates the distribution of peak period capacity or demand over the network intertoll circuit groups by time of day. (The significance of the busy hour is that plant equipment is engineered to meet requirements as to grade of service during that period.) Note that there are many different busy hours during the day, rather than *one single peak,* and that there is a significant percentage of circuits in which the peak periods fall during the morning and evening hours.

The figure also shows the variation in distribution of intertoll circuit group busy hours for the average business day over four selected bands. Note the shift in busy hours by length of haul. This is due primarily to the time zone effect on calling patterns and the large amount of intrastate traffic that is carried on the shorter circuits.

FIGURE 13

DISTRIBUTION OF CIRCUITS BY LENGTH OF HAUL



COMPOSITION OF BASE LOAD

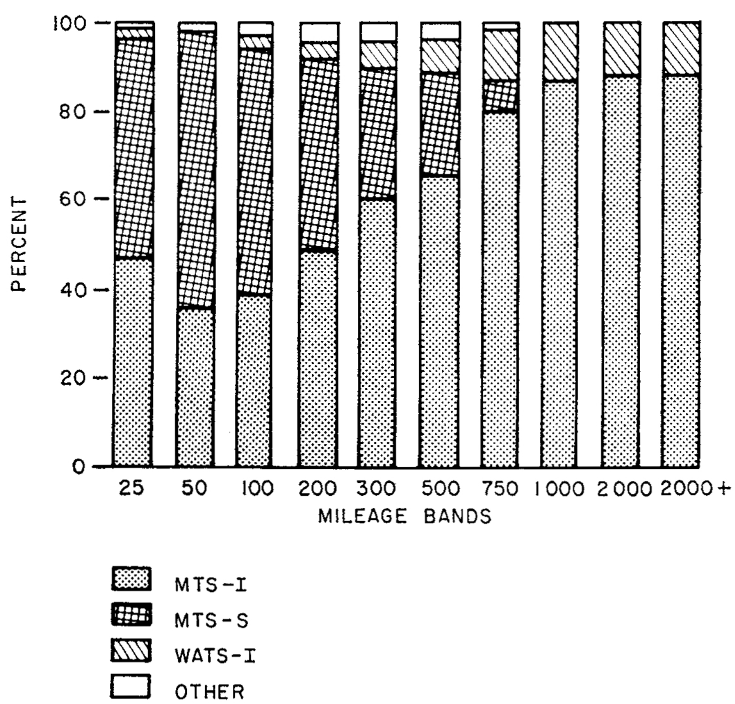
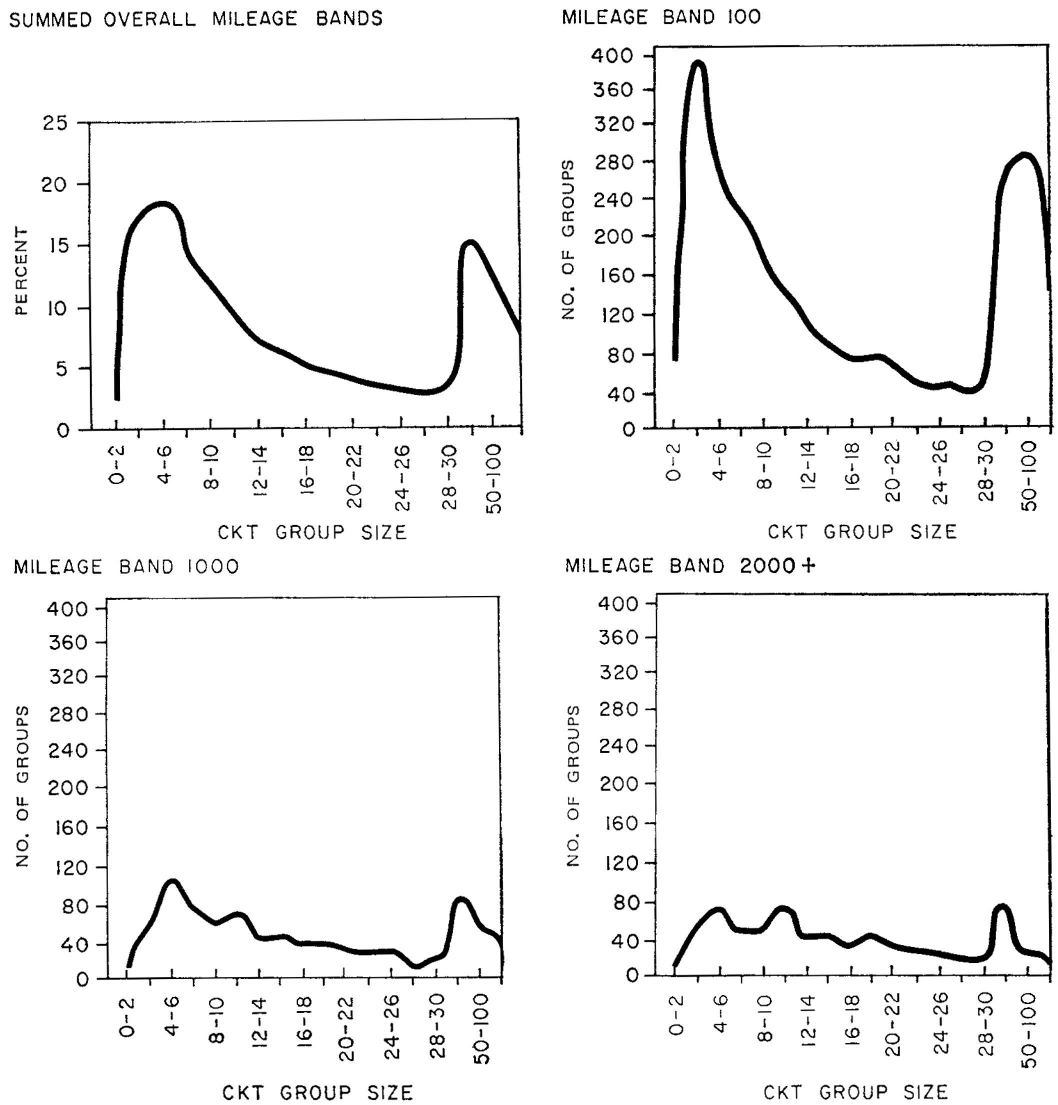


FIGURE 14

DISTRIBUTION OF CIRCUIT GROUPS BY SIZE AND LENGTH



Recalling that networks are designed to take advantage of network economies, and that groups are engineered to serve traffic on a network basis, note the distribution of busy hours for networks shown in Figure 16A and compare it with the interexchange (IX) circuit group distributions shown in Figure 15. The array shown in Figure 16B makes this comparison clear, showing on the diagonal the number of circuit groups which have the same busy hours as their networks. The off diagonal entries give the numbers of circuit groups for which the group busy hour and network busy hour are different. The non-coincidence of busy hours between networks is obvious; it can also be seen that the greatest number of networks have their peak loads during the morning and evening.

FIGURE 15

DISTRIBUTION OF CIRCUIT GROUP BUSYHOUR BY SIZE AND LENGTH

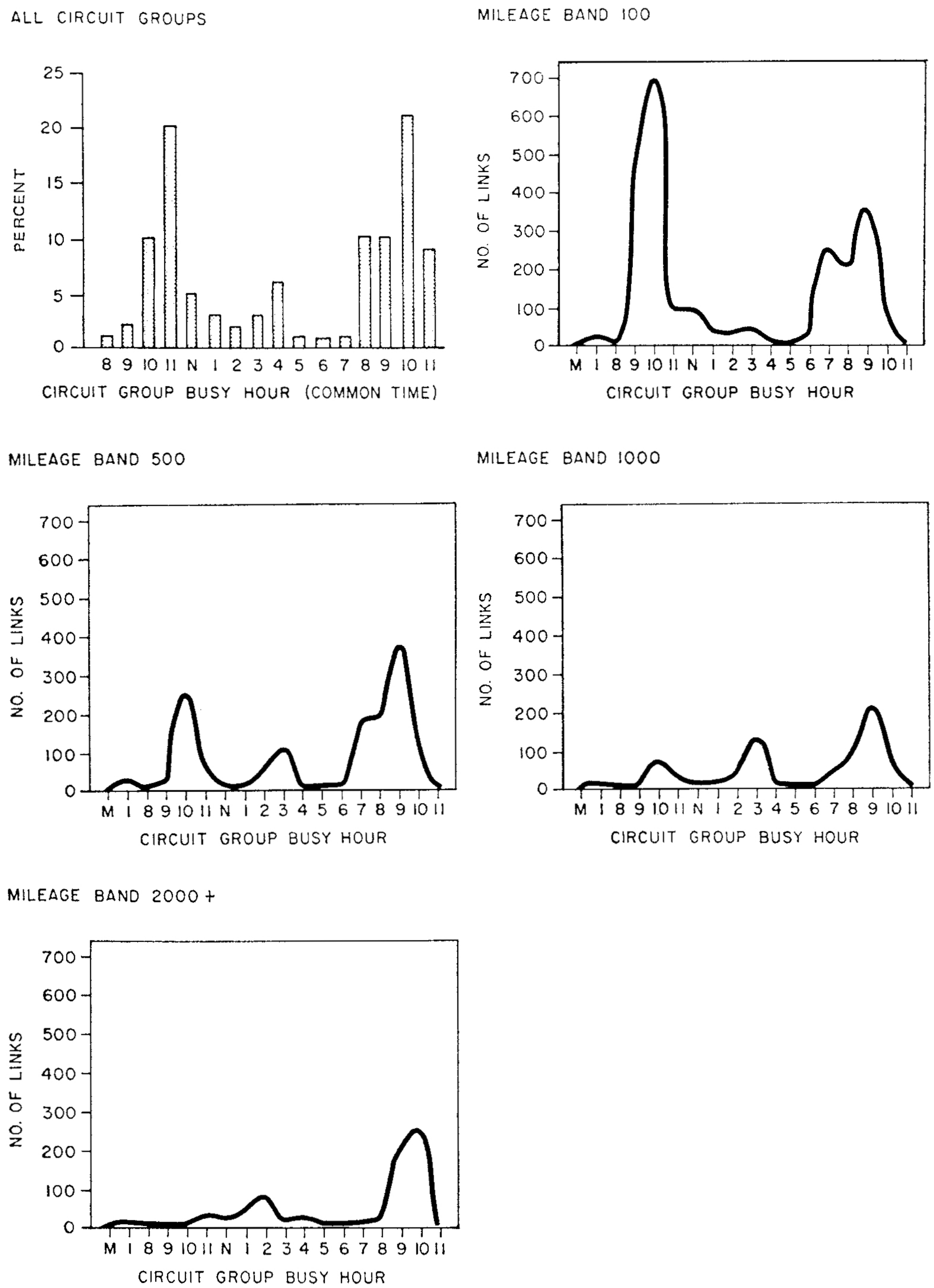


FIGURE 16 A

DISTRIBUTIONS OF CIRCUIT GROUP AND NETWORK BUSYHOURS

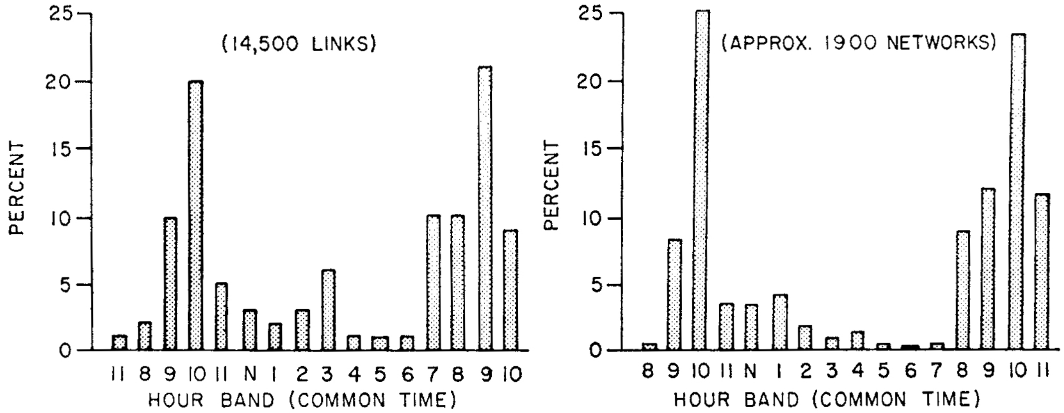


FIGURE 16 B

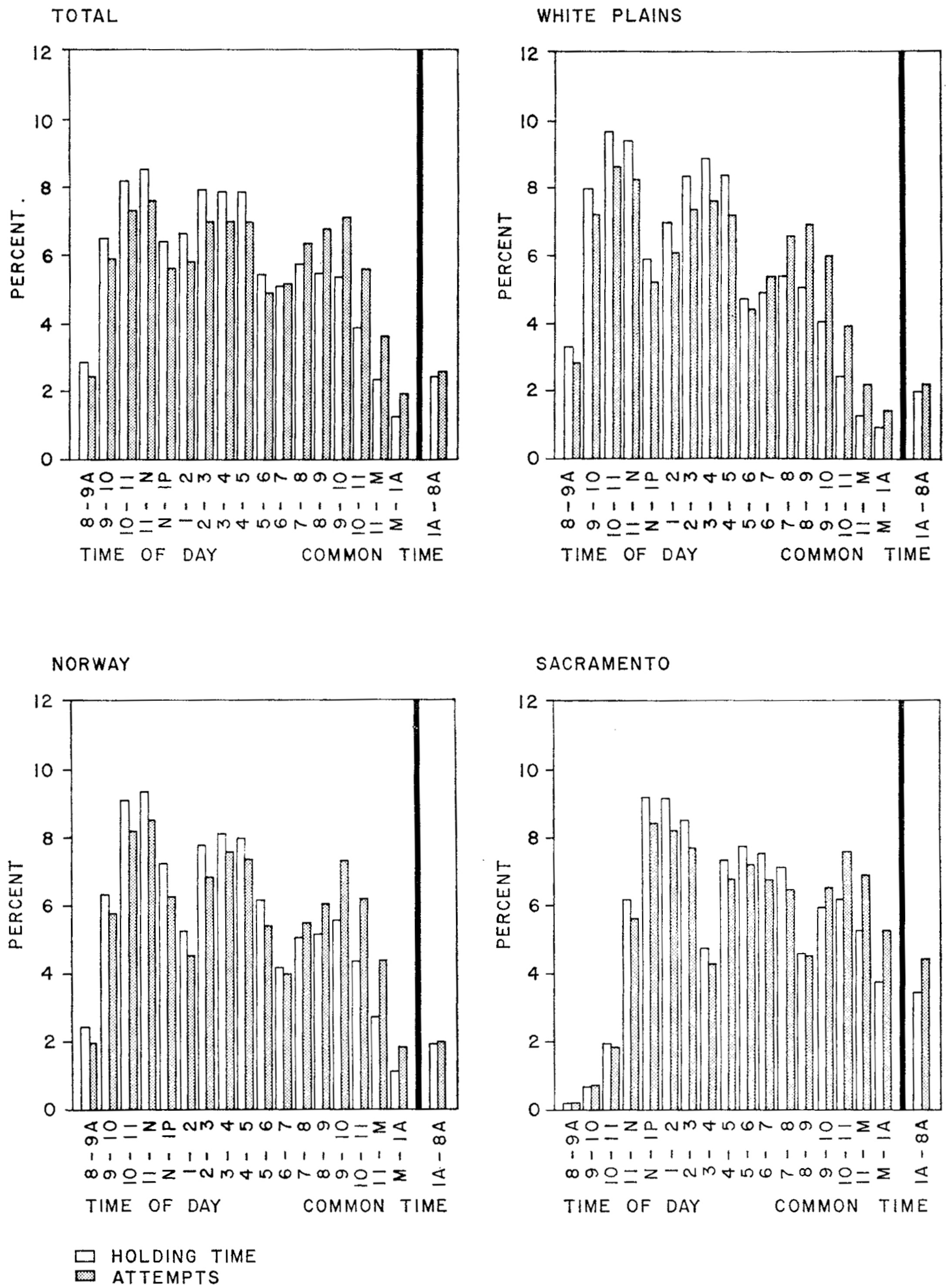
MATRIX OF CIRCUIT GROUP AND NETWORK BUSYHOURS *

NETWORK BUSY HOUR	CIRCUIT GROUP BUSY HOUR																	
NET BH	M	1	8	9	10	11	N	1	2	3	4	5	6	7	8	9	10	11
MIDNT.																		
1																		
8			2															
9		1	7	510	64	7	1	1	10	4	3		19	44	28	32	1	
10		24	6	592	2247	152	11	7	190	314	23	2	29	155	137	282	28	
11		5		13	107	337	24	1	15	64	31	2		3	20	38	15	
NOON		5				7	188	15			1	2				3	31	2
1		4		3	1	1	25	133	8		5		3	1			1	
2				1		3		15	91					2		9	10	
3		1	1	10	68	39	4	1	55	391	12			7	9	19	2	
4					1						6			1		1		
5												1						
6													5	2				
7		2		124	56	5		2	6	6	3		32	829	169	115	9	2
8		9	3	75	97	26	6	1	2		2	1	3	154	549	143	23	
9		7	2	92	191	115	29	5	18	42	18	3	11	210	530	2333	116	5
10	10	89			2	20	94	26	11		8	1			6	115	1072	26
11			1						5				1					2

* Entries in matrix are numbers of circuit groups.

FIGURE 17

DISTRIBUTION OF ABD SWITCHING LOAD



While the circuits are engineered on the basis of number of CCS carried during peak periods of the day, days of the week, over various lengths of haul, etc., switching equipment in the network is engineered on the basis of holding time, attempts, and terminations — that is, the lengths of time for which calls use the equipment, the number of calls which are complete and incomplete (attempts), and the number of circuits terminating in the office. These data are developed by converting the completed messages and related CCS which the demand forecasts produce to total holding time and attempts, a procedure which is described in greater detail in a later section of this book (Translation, p. 76). Since some switching components are sensitive to holding time, others to attempts, and still others to circuit terminations, all three functions of the equipment must be considered when one studies costs associated with pricing changes, if those pricing changes might upset the present set of relationships.

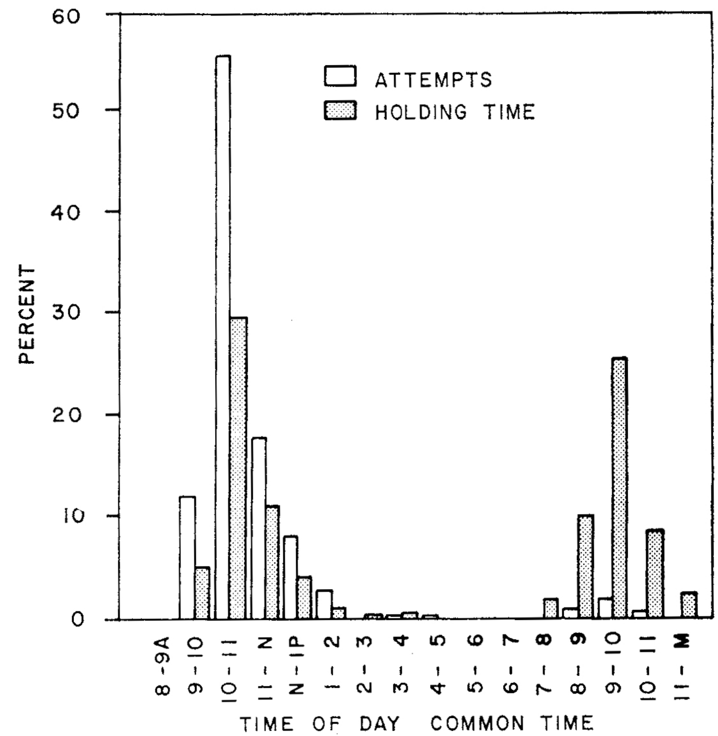
Figure 17 shows the distribution of average business day (ABD) switching load for all regions combined, broken down by attempts and holding time, in percentages occurring within each time period. The distributions for three of the regions are also shown to point up the variations in switching load which exist among geographical areas. Since intrastate prices and price changes are not constant across regions, studies of price relationships must also recognize this variation.

Figure 18 illustrates the comparison between the peak period offered loads used for engineering the intertoll circuit network and the peak period offered loads used for engineering switching equipment.

Figure 19 illustrates, for three regions separately and then for all regions combined, the distribution of the switching load busy hours by toll center areas (TCAs). These busy hours also are different for holding time and for attempts, and — recalling that switching equipment is required to perform three functions (control, terminating, and holding) — one can readily see that an economic capacity for switching must be determined on the basis of the various peak load distributions. It is equally important that a research facility for investigating price cost relationships be founded on a recognition of inter- and intra-TCA peak load distributional variations.

FIGURE 18

DISTRIBUTION OF BUSYHOUR BY TCA (SWITCHING EQUIPMENT)



The traffic operations associated with providing telephone service require specific attention in the formulation of any communications supply model. The load offered to traffic offices, in which operators and switchboards are located, determines the work force required to provide service. Figure 20 is a series of graphs showing the distributions of traffic work units and how they vary over some sample geographic regions. Traffic work units are a way of representing the traffic office load, and are used to measure the work which is required on the part of operators at the boards. The distributions in the figure are given as a combined regional total and for three regions, showing intrastate load as the white areas and interstate as the shaded. It can be seen that the traffic work load varies not only over the average business day but between regulatory jurisdictions.

Figure 21 illustrates the total (all-regional) and regional distributions of numbers of operator positions in chief operator switching units. The three regional distributions shown are particularly helpful from an analytical standpoint because they point out the variations in distribution over geographic areas.

FIGURE 19

DISTRIBUTION OF BUSY HOUR BY TCA

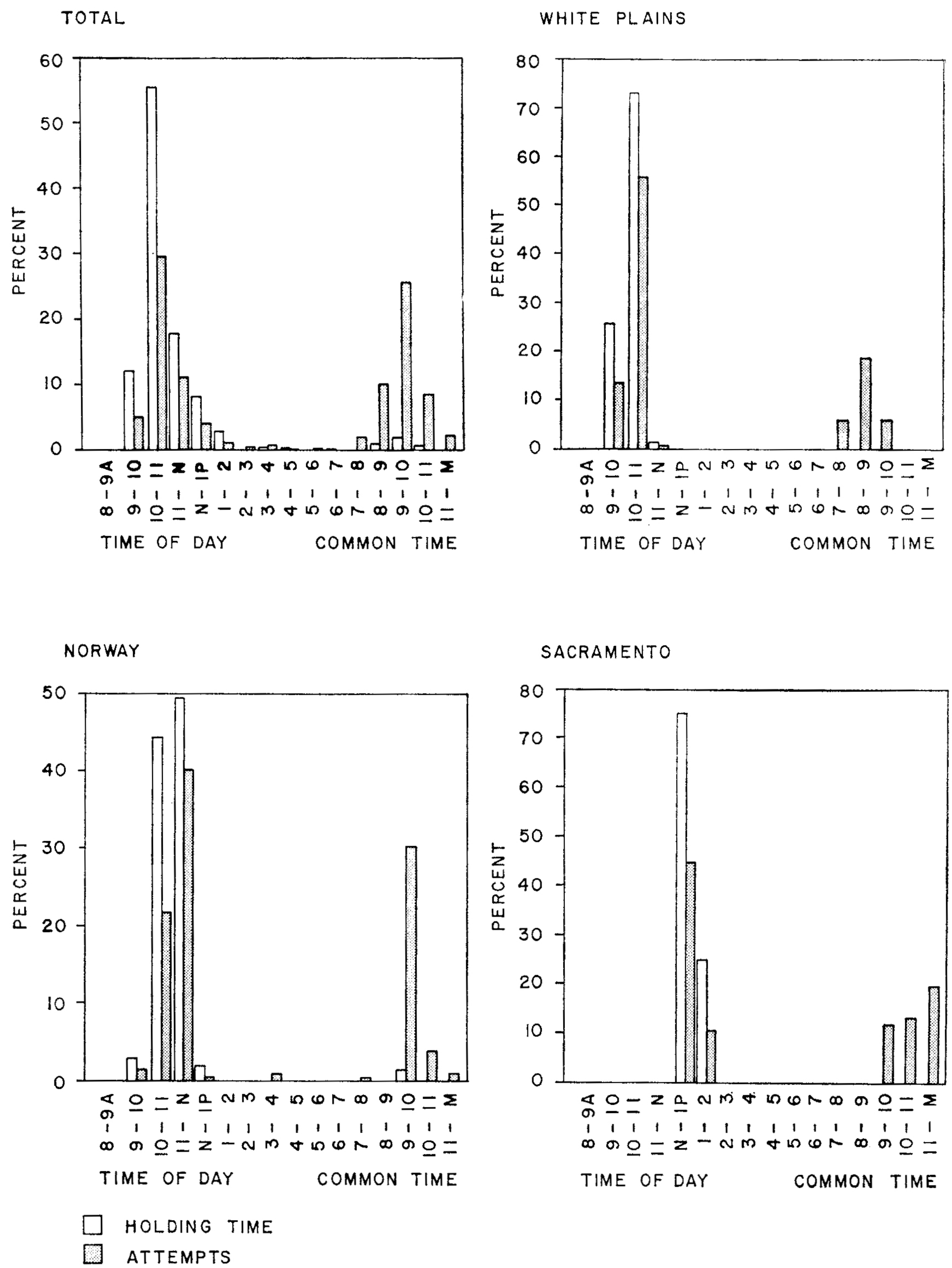


FIGURE 20

DISTRIBUTION OF ABD TRAFFIC WORK UNITS

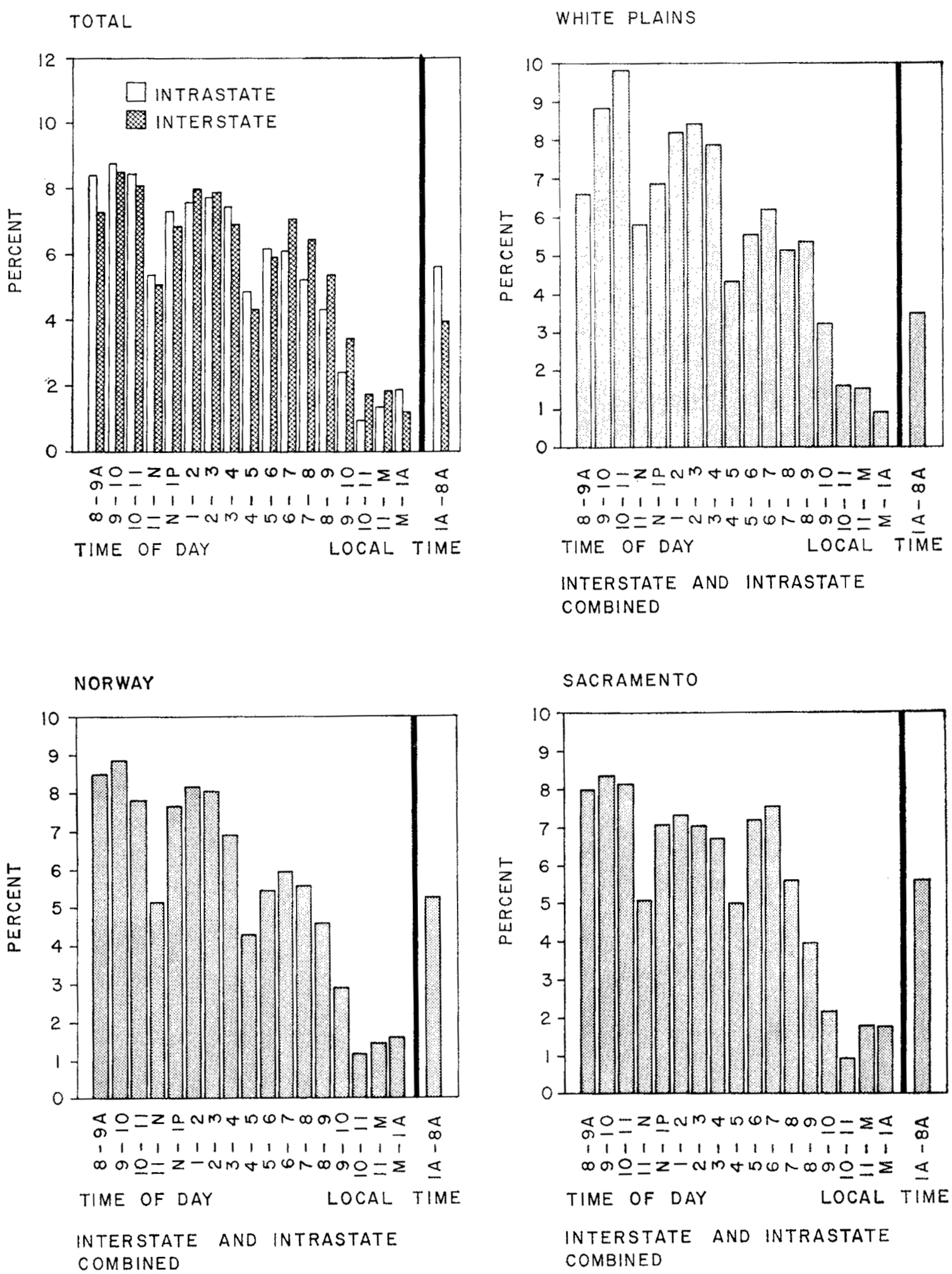
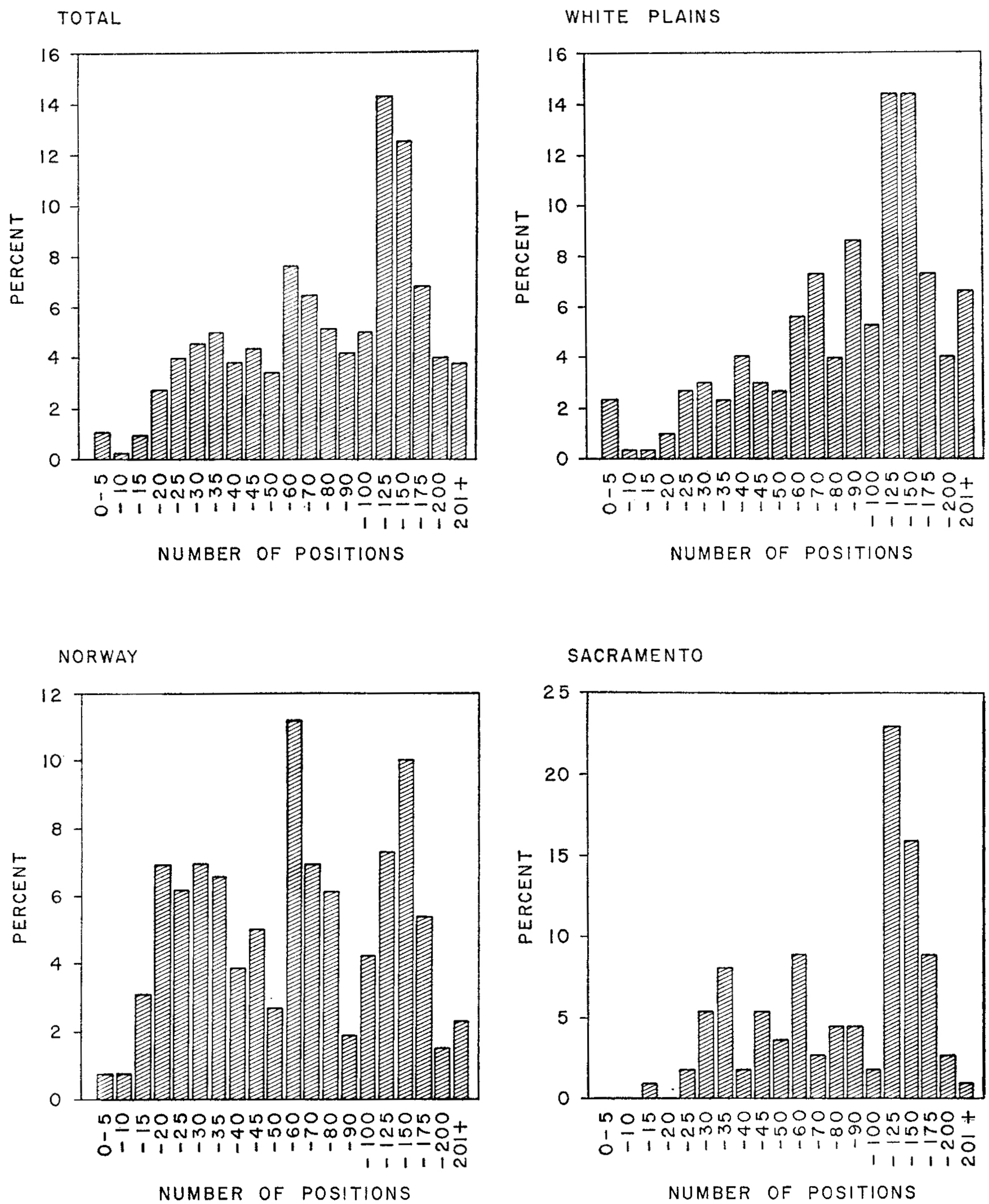


FIGURE 21

FREQUENCY OF POSITIONS PER COU FROM TRAFFIC BASE



6. Economic theory related to an operational model

Building a facility with the capabilities described in the foregoing is not easy. Economic theory has not yet been developed and coupled to an operational procedure which can handle the complexities associated with modeling a multi-product firm, in which plant is used in common, where there are joint products, where substantial economies of scale are possible, and where demands on the supply system vary in time and space so that peak-load pricing schemes offer advantages.

Views differ as to how costs should be assigned ideally to products (services and sub-classes of service). However, an understanding of the principles and methods actually in use in costing practices is essential if the system is to be modeled in a useful and valid way. Costing procedures provide a basis for studying pricing practices used in the business, and they also provide the basic mechanism through which economic efficiency, subsidization, and other related effects can be investigated.

Relationship to cost is only one consideration that bears on the setting of prices, and there are many other aspects which regulated firms must consider. Some of these are:

(a) Continued inflation,

(b) Availability and cost of money,

(c) Regulatory attitudes and decisions,

(d) Changes in the technology, and

(e) Environmental matters, including the prevailing political and social temperaments.

There are socio-political arguments which favor keeping some sort of average pricing in many situations in order to support increased investment. The availability of extensive communications services, for example, might yield benefits for the nation that would not necessarily be realized in the mere satisfying of the short-term demand for telephone service. Arguments have also been advanced in favor of encouraging the introduction of new technology for its own sake. These two positions imply that prices should be set to encourage rapid growth, perhaps even at the cost of preserving the cost-price relationships held to be desirable in theory by some economists.

However, pricing cannot be considered an exact science based on economic factors alone. There are certain general principles that are useful for guidance in investigating utility price changes. These are:

(a) Changes in price should at least take into account changes in cost for the specific individual services or categories of service to which they relate.

(b) The company's overall rate of return should be sufficient to attract the necessary capital to the enterprise.

(c) The rates charged for separate categories of service should be structured in a manner that will result in resources being used efficiently, thus helping to minimize the total cost.

(d) The rate structure of the various services should encourage customers to buy service that meets their needs and makes effective use of the plant investment, thereby once again helping to minimize costs.

(e) Although rate changes apply to the future, in any rate review the utility must start with the existing rate structure. Adjustments can then be formulated as iterative steps toward a set of target rate structures.

(f) Current and prospective costs, as opposed to historical costs,[4] should be treated as the relevant economic costs for all decision making purposes.

Dr. James C. Bonbright has stated in connection with a recent FCC investigation[5] that, in the determination of service rate levels, the relevant costs of supply for any specific service or class of service are incremental costs (usually long-run). He points out that rates cannot be set at these incremental costs for the telephone industry, since the resulting total revenues would fall short of meeting overall revenue requirements. However, they are useful in setting minimum rates or class rate levels. Actual rates would exceed these minima by mark-ups sufficient, in the aggregate, to yield total revenue requirements.

Defining incremental costs. In most public utilities, total capital costs are very large. With regard to utility cost economics, it is not immediately apparent what is meant by or what constitutes "incremental" or "marginal" costs. Before one can even attempt to measure them, a number of difficult economic questions have to be answered with respect to what it is, precisely, that is being measured. This difficulty is recognized when we consider the joint or common use of facilities by the various services which the Bell Systems offers, the complex network structure which these offerings necessitate, and the non-dedicated nature of the facilities of the network. One must add to these the generic difficulties associated with analyzing service offerings (as opposed to products) in any economic system. Let us begin by defining some aspects of the problem.

A. The size of the demand increment

A major element of the incremental demand-cost definition problem is the size of the demand increment, especially since the level of incremental cost per unit will depend on that value. The larger the increment or unit of service or demand, the more costs become variable. To explain this fully, it is helpful to examine the service space partition, *i.e.*, the interrelationships and categorizations of service offerings.

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[4] Transcript of testimony given in FCC Docket 16258 by W. Baumol, pp. 14021 ff.
[5] FCC Docket 16258.

TABLE 4

MAJOR BELL SYSTEM SERVICES

SWITCHED NETWORK SERVICES			
INTERSTATE TOLL	INTRASTATE TOLL	LOCAL	NON-CHARGED
MTS WATS OUT WATS IN DATAPHONE PICTUREPHONE	MTS WATS OUT WATS IN DATAPHONE PICTUREPHONE	FLAT RATE MSG. RATE EXTENDED AREA BUSINESS VERTICAL SPECIAL	INFORMATION OPER. ASSISTANCE INEFFECTIVE ATTEMPTS

Table 4 summarizes the various major network service categories. As can be seen, service classifications are first nested within each of the regulatory jurisdictions. Note the category for which no charge is made to the customer, which in reality is nested within each of the interstate, intrastate, and local services. Within each service classification is a hierarchical or nested structure consisting of a number of categories for which different prices exist. Figure 22 shows how one of the services (long distance interstate toll) is further structured, by partition of the price space, according to the following categories:

(a) Class of call (DDD, operator-station, operator-person),

(b) Day of week (weekday, Saturday, Sunday),

(c) Time of day.

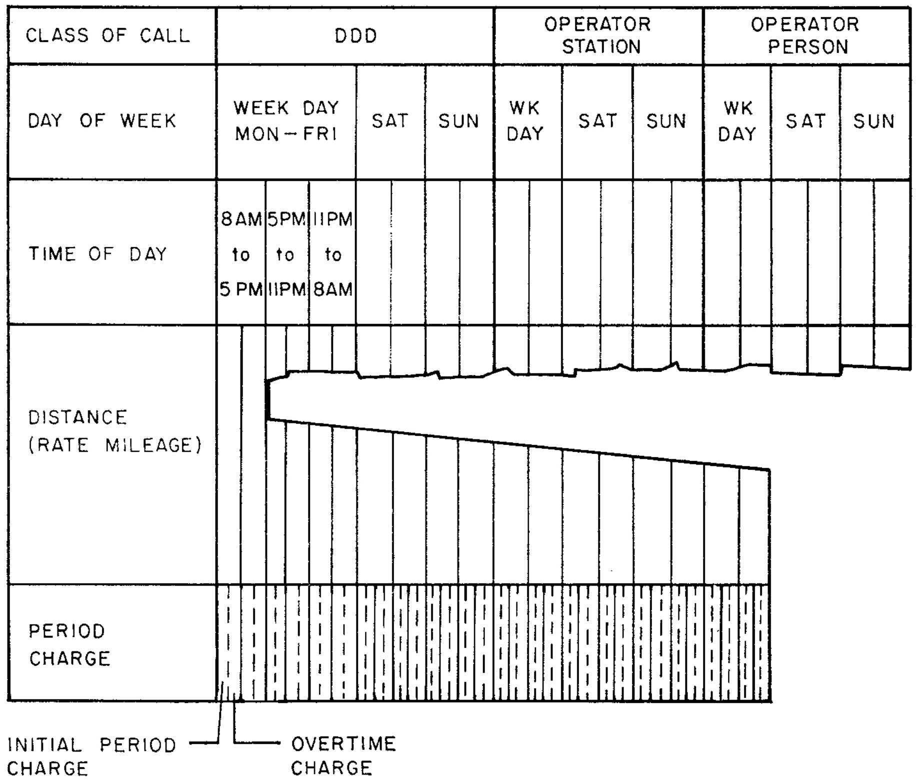
(d) Distance (length of haul), and

(e) Initial or overtime period.

In defining a particular segment of demand, it should be observed that certain classifications are statistically crossed (that is, class of call/day of week) while others are statistically nested within those categories (time of day/distance/period charge).

FIGURE 22

INTERSTATE MESSAGE TELECOMMUNICATIONS SERVICE (MTS)



In defining a demand increment and the associated incremental costs for evaluating and studying pricing relationships, the problem thus becomes one of estimating the various increments associated with the specific categories as well as the total increment for a particular service. Size of increment becomes important if one attempts to define unit costs. Many economists recommend that prices be determined systematically by setting them equal to some sort of unit costs, *i.e.*, marginal or average costs. These advocates of marginal-cost pricing base their arguments on what is called "welfare economics," in which the goal is to maximize some "welfare function." This function depends in general upon state variables, including each individual's consumption of each good. In order to derive any pricing relationships, welfare economists have found it necessary to make rather stringent assumptions. Typically, they assume that:

(a) No indivisibilities exist,

(b) No other "externalities" (such as pollution) exist which would cause firms to affect each other or the consumers outside the marketplace,

(c) Each consumer and each corporation has complete information on all possible alternatives, and each behaves rationally,

(d) Technology is held constant, and

(e) Expectations play no role in the choices made by individuals or by the firm.

Since the way the switched network is designed and structured in the Bell System contradicts many of these assumptions, principally with regard to the theory of economies of scale, this theory would be of only limited use in the structuring of a suitable cost definition for the model. Instead, the analysis develops average incremental costs associated with the increments of demand to assist in the exploration and evaluation of alternative cost and price principles. However, these must be used with caution, and the size of increment is significant in their interpretation.

B. The time perspective

Throughout the analysis, the relevant incremental costs will be considered to be those incurred in the long run. Besides covering direct short-run costs of labor and materials and a normal rate of return on additional capital, these costs will include amounts sufficient to cover capital costs required to provide the incremental units of output over a period in the "indefinite" future.

The costs that are relevant in developing economically efficient pricing schemes for the future in firms where plant is already constructed, with production capacity already installed, are the variable costs of operating the firm. The longer the time frame, the greater the proportion of costs that become variable. Interacting with the time perspective is the phenomenon of decreasing costs which characterizes most utilities. Since costs are sensitive to that parameter, selection of the appropriate time frame is important for measuring or estimating incremental costs.

Consider first the selection of a short-run time frame for determining short-run incremental costs. Once capacity investment is made, output can be increased with declining costs until the physical capacity of the facilities is reached. The only variable costs are those associated with operating at existing capacity. As the time perspective increases, the costs of repairs, maintenance, and operation can also be included.

When the time frame is being selected, it is also important to consider the effects of technological progress (which can occur over the short or the long run, but which may result in increased costs in the short run and in decreased costs in the long run). The "long run" over which cost outlays are determined for a model using a long time frame is usually the period equivalent to a planning period for capacity installation. If a different period is selected, *i.e.*, one shorter than the planning period, there is a risk that all costs associated with providing service may not be Included. Capacity is added to the telephone network where needed, usually with the expectation that the addition will be fully used within four to six years. Prices for telephone services are therefore usually based on the estimated costs for a four-to-six year period, which is the probable interval between rate adjustments.

However, the time horizon parameter in this model is variable, and is an option of the analyst using the model. The planning period is usually held constant, as is the relative technology when the various alternative price schemes are compared or evaluated.

C. Identification of costs shared in common

Another difficulty encountered in defining and computing incremental costs associated with a demand change for a given service or sub-class of service is the association of costs with that particular service classification, in an environment where the major portion of the plant used to furnish network services is used jointly by all services and sub-classes of service. Since these categories of service can vary in the proportion of plant which they use, separate marginal production costs can be computed for each. However, the problem is to measure or estimate in a meaningful and accurate way the separate marginal costs associated with simultaneous changes in the demand for so many different products (services or classes of service).

D. Nature of non-linearity of incremental costs

While it is obvious that some form of economic returns to scale may operate within the network, the nature of non-linear behavior of the system introduces other considerations. The usual consequence of non-constant returns to scale is that classical marginal-cost pricing does not necessarily cover costs, nor necessarily result in optimal allocation. These effects are important, but they may in principle be roughly compensated for at an appropriate level of aggregation by some proportional adjustment procedure. The usual notion of non-linearity which motivates the returns-to-scale argument in economics is one of smooth functional variation. The classic textbook example is simply the non-linear relationship between area and circumference; a firm with twice the floor space does not require twice as long a fence around its building. This kind of returns-to-scale effect must operate within the network services, but it is of relatively minor importance. This is partly true because such non-linear behavior is necessarily locally very smooth, nearly linear. It is only when very large changes are considered that such functions could play a major role.

However, the switched network facilities contain another kind of non-linearity which we term "strong non-linearity." Strong non linearity is defined as non-linearity which cannot be approximated locally by a linear, or any smooth, function. One indication of this form of non-linearity in the network is the fact that substantial load increases on the system can in certain instances result in a substantial decrease in the equipment requirements, and therefore in costs. Equally, load decreases can result in increased equipment requirements. Finally, total load can be held constant, and only the relative timing of the load during the day varied. In this case incremental costs may increase, decrease, or stay constant. Thus, if a price change or other inducement causes some subscribers to change the timing of their telephone use this may have a large effect on costs. However, the sign of this effect is, a priori, unpredictable.

Strong non-linearity in the network stems from the peak load engineering practices which determine equipment requirements as a function of the load at the network busy hour. These practices follow directly from the objective of providing a specified minimum level of service at all times. They imply that circuit loads can be increased during off-peak periods with very little effect on long-run incremental costs. In addition, an increase in load which also induces some change in the timing of calls may result in a decrease in network traffic load at the busy period. This will produce the situation in which total load increases

while incremental costs decrease. Significant effects of this kind are observed in practice and in use of the model.

The smooth form of non-linearity which leads to non-constant returns to scale would suggest that aggregate cost and supply functions would have to be handled with care so that the range of valid aggregation was not exceeded, but the aggregations would be locally valid. This is not true of strong non-linearity. Even a minor perturbation of demand can cause changes which could not be predicted at all at the aggregate level. This characteristic of the network led us to a highly disaggregated analysis of the load-carrying capacity of the network, and to careful consideration of the nature of demand changes.

A related problem on the demand side is the fact that the behavior of demand cannot reasonably be represented in the classical framework of many independent consumers. It is this independence that is required for any meaningful interpretation of the usual economic proposition that a decrease in supply leads to an increase in price. For the standard commodities of classical competitive economics, price approaches a maximum as the quantity approaches zero. If there were only one telephone available on the network, its price would be rather low. More generally, it is clear that a major part of the value of the telephone is the opportunity to place a call to many different points. Such factors as these have been very little studied within economics. [A remarkable exception is Nyblen (See Reference [6]).] What study there has been shows immediately that standard techniques of aggregation are entirely inappropriate and misleading. This was further incentive for us to take a disaggregate approach in the model areas where it was clear that strong non-linearity entered in an important way.

The above does not imply a wholesale rejection of aggregation. We did, in fact, apply aggregations in many areas. Sometimes this resulted from a lack of data or a lack of analytical understanding. But in many other cases it was clear that the extent of non-linearity or interdependence was relatively minor, or the total effect of that particular component was sufficiently small so that aggregation would provide sufficient information.

E. Average incremental costs

A practical, achievable benchmark is required for studying price cost relationships. Average long-run incremental costs provide such a measure for evaluating pricing schemes when related to expected increments in demand. Average measures are required since demand is constantly changing and cost functions are constantly shifting. Therefore, long-run incremental costs will be based on average incremental variable costs associated with increments of demand estimated as averages over a planning period extended into the future, to allow for the variations in demand and the associated shifts of the cost functions. This requires estimating the additional capacity costs that will have to be incurred in order to sustain continued growth at given prices.

AT&T computes costs associated with providing interstate toll service on a nationwide basis, because of the manner in which such service is provided. Communications paths are switched automatically from one route to another, using whatever circuits are available and taking advantage of the excess capacity on some routes. Average costs are therefore

considered a practical way of accounting for local differences in capacity for pricing purposes. The individual operating companies, in estimating costs for intrastate toll services, use the same average cost concept, based on geographical areas associated with jurisdictional boundaries. The separations process plays an important role in this concept, for computing the relevant rate basis.

Economic theory defines incremental costs as the sum of all costs that vary with output. In the case of a regulated utility, operating subject to a profit constraint or an explicit rate of return constraint, incremental costs will include capital investments, operating expenses, income taxes, and a fair rate of return allowed the utility.

Therefore, since the average long-run incremental costs will include all variable costs, they will include incremental plant and equipment costs associated with a change in demand for facilities that are needed at present and in the future. They will also include operating expenses resulting from an increment in demand. In a growing firm, the volume of plant needed will change over time; the incremental investment figures used for long-run modeling should therefore refer to average deviations from this growth pattern. However, the total average cost figures will omit certain cost components which are not affected by demand; such values might be capital costs arising out of past investment decisions, which will remain part of the company's outlay regardless of the volume of sales.

F. Discounting of future costs and future outlay

Since changes in demand will involve future values, to put all of these on a basis so that they are meaningful for comparison purposes, they must be "discounted" in some manner, *i.e.*, present value calculations techniques must in principle be employed. Therefore, average long-run incremental costs in terms of incremental capital costs for comparison purposes will refer to discounted present values of the stream of expected outlays.

That is, we attempt to visualize the series of outlays, their magnitudes, and the dates at which they will occur, each discounted by the appropriate rate for translating them to present values. The rationale from the economist's point of view is that if a decision is made, that decision is going to bring in a stream of revenues and will also obligate the firm to incur a future stream of costs; in deciding whether or not the decision is an economic one, the firm must discount the future stream of revenues and costs and compare the two. Thus, to determine and evaluate the price-cost interaction in order to choose between alternatives requires that cash flow procedures be applied to the stream of cash flows, to convert them to an aggregate comparable basis.

This raises the question of the correct rate of discount to be used in the process. It is usually accepted that an appropriate discount rate is the firm's so-called cost of capital, and that the firm's permitted rate of return is generally determined by procedures designed to get at that figure. Difficulty arises when one attempts to evaluate whether or not the cost of capital or allowed rate of return are dependent on the investment program. For this analysis, the cost of capital (or rate of return) is exogenous and is held constant in the evaluation of alternative pricing schemes.

G. Test year concept

The estimated capital outlays, expense outlays, and revenue streams that are derived by the model are translated into estimates of a form normally used for annualization of data supplied in support of rate proceedings. The normal practice is to estimate demand, revenues, and revenue requirements for a test year on an annual basis. In this analysis, estimates for a test year are derived in the following manner: first, average long-run investment streams are developed and dis counted to present values; then average, forward-looking depreciation rates (developed external to the analysis) — *i.e.*, rates which reflect future depreciation policies — are applied to the discounted investments by class of plant, giving estimates on a test year basis. The estimates obtained by this procedure are considered average long-run incremental depreciation expenses for the test year. (This process is discussed in the next section in more detail.)

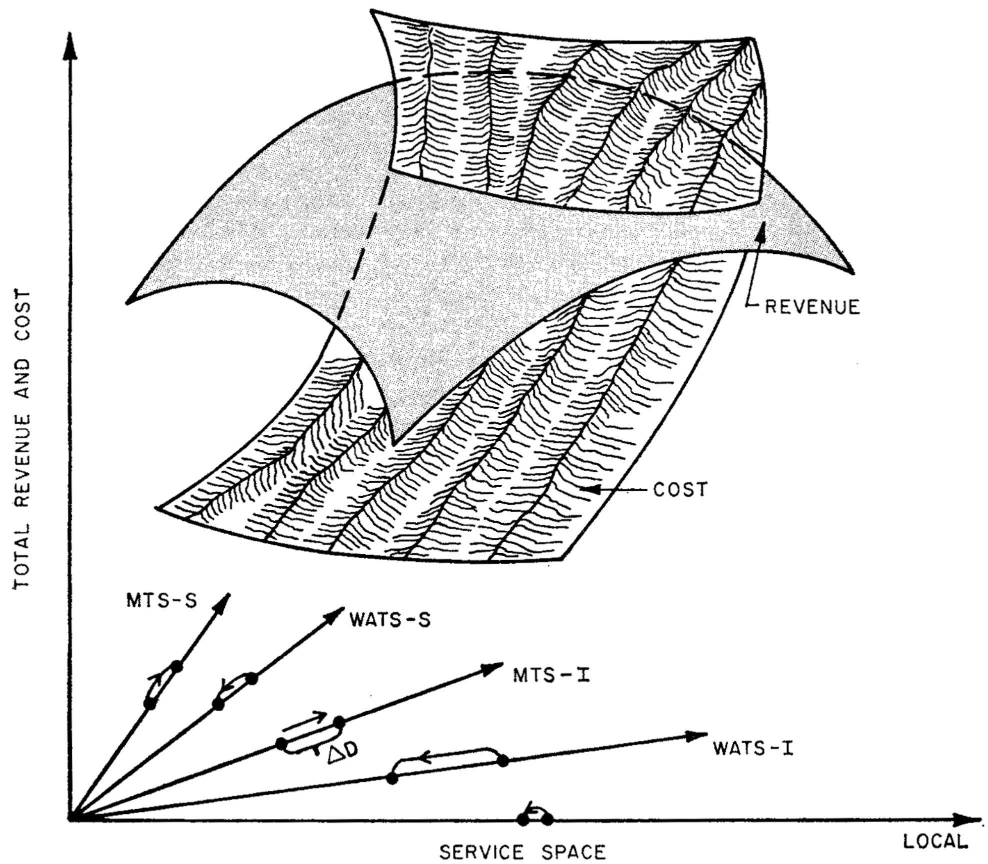
The operating expense estimates are computed on an annual basis for an average annual increment of demand, and reflect average future labor rates. This assumes that the growth rates do not change as a result of a price-demand change. The revenues are also derived on an annual basis through application of essentially the same assumptions. (These too are discussed in more detail in the next section.)

Test year annualized estimates are finally developed by inclusion of the estimated operating expenses (including depreciation expense), income taxes, and a fair rate of return. (In a later section, entitled Annualization of Costs and Revenues, additional details are given. See p. 162)

The total revenue — total cost methodology. To estimate long-run incremental costs for particular segments of demand, a measure was developed based on total revenues and total costs. The conceptual abstraction shown in Figure 23 illustrates the transformation of the major network services to cost-revenue relationships, in order to structure the service space for economic analysis. The service offerings are shown along the abscissa as a set of vectors, in the sense that the arrows are the vectors that result when all of the components of each offering have been combined. (Recall that the MTS service structure has a number of pricing components.)

FIGURE 23

TRANSFORMATION OF MAJOR NETWORK SERVICES TO COST-REVENUE RELATIONSHIPS



On the ordinate are the single-dimensioned (and observable) quantities, total revenue and total costs. Within this space, we have the n-dimensional revenue and cost hypersurfaces[6] which depict the transformation of service to revenues and costs. These hypersurfaces represent the network transformation mechanism (or production function) for each level of service offered.

The service vectors are shown to be non-orthogonal, to reflect the interactive and dependent nature of service offerings and levels of service. For example, if the demand for MTS-I (interstate) shifts by the amount ΔD shown, that shift may induce changes in all levels of the other service offerings. The overall change will then be observed as a shift in total revenue and total cost (a one-dimensional point-to point shift) on the revenue and cost surfaces. Therefore, a Δ change in demand produces a new vector that in turn gives a new total cost and revenue, which we will refer to as a point-to-point shift.

[6] Empirical work has indicated that these surfaces are not smooth but have creases, folds, and significant discontinuities.

Determination and delineation of these shifts and of their effects are the objectives of the model and analysis being described in this work. Only at this level of detail can incremental costs be associated directly with service offerings and their nested categories, thus providing a sound basis for pricing practices. The detailed structure of the model allows this analysis to be performed in an operational manner.

Now consider a two-dimensional projection of this service space; *i.e.*, consider the surface that can be viewed by slicing down the ordinate and out along the vector MTS-L This surface is shown in Figure 24, and is similar to the standard economic total revenue — total cost approach taken in traditional pricing theory. In fact, it was this similarity to traditional concepts that suggested the approach finally taken in the model. However, some significant differences between the classical theory and the Bell System adaptation need to be emphasized.

First, in the traditional total revenue-total cost graph, the abscissa is a scalar, while here the abscissa is a vector. For Bell System modeling purposes, when a change in service demand is made, the resulting cost or revenue changes will not be simple but will involve many interaction terms which are the most informative and essential parts of the analysis.

The "curves" in the graph should not be viewed as simple functions of a single variable, as would be true in the classic case. These curves are two-dimensional projections of multidimensional functions; any movement along the curves will therefore not always be simple in nature, but will represent, for certain cases, shifts of the hyperplanes, which will cause new dimensions to appear in the projection. (Again, it is the interaction terms between offerings and service levels which cause this.) It is therefore difficult to determine at any particular level the shape of the total cost-total revenue curves; only the points R_1, R_2, C_1, and C_2 can be known. However, through applying this knowledge to the model, a rich body of insights derived from individual incremental quantities can be gained.

FIGURE 24

TOTAL REVENUE - TOTAL COST APPROACH

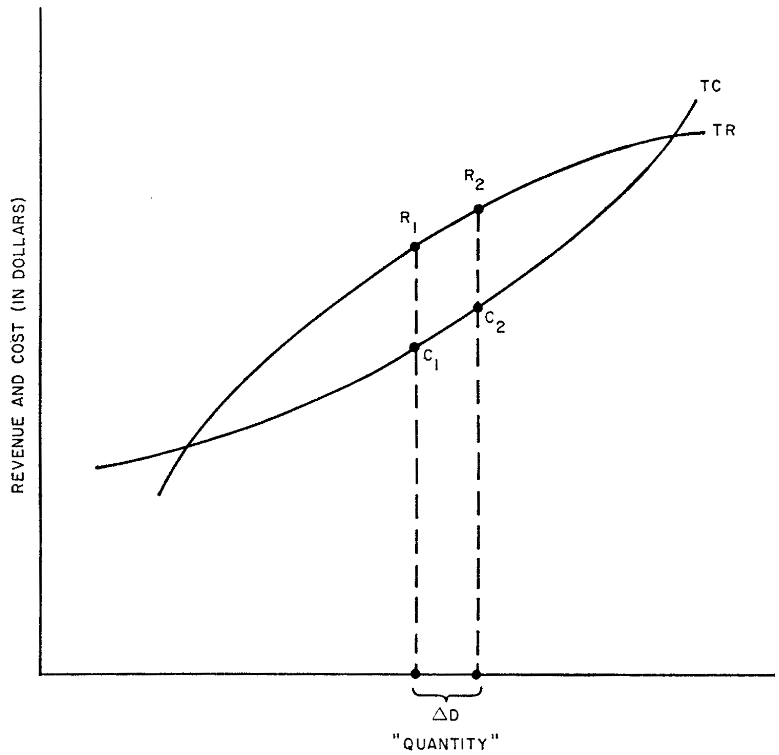


TABLE 5

METHODOLOGY

MARGINAL	INCREMENTAL
PRODUCT = q	PRODUCT = "SERVICE"
COST = C (q) + b	COST = C (NETWORK)
REVENUE = R (q)	REVENUE = R (NETWORK)
MARGINAL REVENUE = $\frac{\delta R}{\delta q}$	INCREMENTAL REVENUE = $\frac{\Delta TR}{\Delta D} = R_2 - R_1$
MARGINAL COST = $\frac{\delta C}{\delta q}$	INCREMENTAL COST = $\frac{\Delta TC}{\Delta D} = C_2 - C_1$

There is still another aspect in which this model differs from the classical case. With the more familiar techniques of analysis, cost revenue interaction would be looked at sequentially by services, and the effect on each service would be viewed independently. The abscissa,

as we have said, would be a scalar, and the curves would therefore be functions of one variable. With the classic approach, a graph would therefore be generated to show cost-revenue changes in total for each service. In the methodology being described, the abscissa is a vector, and the cost-revenue curves are therefore projections. These demonstrate changes in the operating points R and C, and permit the curves to change in form and shape as new dimensions of the interactive effects become prominent.

To summarize, the methodology developed for this model represents an attempt to generalize in an operational sense the standard single-variable, total-cost, total-revenue approach. In addition, the adapted technique provides a means of analyzing the complex case where there are service offerings instead of products, levels of service instead of quantities of output, and service differentiations defined such that "production costs" can only be indirectly related to "products."

Table 5 compares the classic, marginal-cost case with the incremental-cost methodology. The significant difference is that the results of the incremental-cost analysis show total incremental costs and total incremental revenues, which appear to be more meaningful (because of the indivisibilities of the Bell System offering space and the network and cost spaces) than the classic results showing marginal costs and marginal revenues.

Figure 25 illustrates the representation of the functional relationships within the multi-dimensional cost surface by analyzing total network costs. When the incremental costs are computed, all major inter active terms are considered and combined to produce the totals shown. However, in this combination the interactive effects are not lost; each can still be examined in terms of both its magnitude and its sign. In using this model for the analysis of price-cost relationships, the unpredictability of these interactions has been revealed in many actual analyses. What has been shown is that the curves of the projection shift in space, but the nature of the total cost function is as yet not clearly understood.

FIGURE 25

NETWORK TOTAL COST FUNCTION

$$\text{TC (NETWORK)} = \text{C (OPERATING, CAPACITY, ...)}$$

$$\begin{aligned}\Delta\text{TC (NETWORK)} &= \sum_i \Delta C_i\,\text{(CAPACITY)} + \sum_i \Delta C_i\,\text{(OPERATING)} \\ &+ \sum_i \sum_{\substack{j \\ i \neq j}} \Delta C_{ij}\,\text{(CAPACITY)} + \sum_i \sum_{\substack{j \\ i \neq j}} \Delta C_{ij}\,\text{(OPERATING)} \\ &+ \sum_i \sum_j \sum_k \Delta C_{ij}\,\text{(CAPACITY x OPERATING)}_k \\ &+ \text{HIGHER ORDER TERMS}\end{aligned}$$

RESULTS OF ANALYSIS YIELD

$$\Delta C_{\bullet}\,\text{(CAPACITY) AND } \Delta C_{\bullet}\,\text{(OPERATING)}$$

Investigating utilization of plant resources. The problem of plant utilization is the final question which must be discussed relative to defining and computing incremental costs

that will assist in studying the Bell System's price-cost relationships. If the same type of plant capacity were to serve all users, then capacity costs could be levied only on utilization of plant at the peak load. However, Alfred E. Kahn[7] has claimed that capacity costs ought to be shared in varying proportions from 100 percent down to zero, by purchases of plant during periods that have varying likelihoods (corresponding to the percentages of cost shared) of becoming peak periods during the foreseeable future. The views of economists vary as to the assignment of peak load responsibility. One opinion states that, in order to investigate the effects of pricing (which include utilization of plant during various periods of the day with changing demand) it is necessary to estimate incremental costs for various segments of the rate structure (and related market).

There are numerous methods that can be used to allocate or assign incremental costs to specific segments of the market, usually involving the assignment of costs on a peak-period responsibility basis. (The rationale is that capacity must be constructed to serve the peak load, whereas off-peak loads can be treated as incidental to the problem.) A traditional criticism of most peak-period procedures for allocating responsibility for costs is that they usually fail to account for changes in the peak, which may entirely alter allocations; a result of this failure can be highly unstable and variable costs. Since non-peak periods may become peak periods, it is necessary to assign an appropriate weight to the demand recorded at times when the system is not at its peak in order to avoid drastic changes in allocations of costs in the future. Only a portion of the cost of capacity constructed is directly attributable to an increase in peak demand.

To eliminate the peak-period allocation problems, costs are sometimes evaluated on a distributed responsibility basis. Under this procedure, each rate class (and users subscribing to that rate class) bears the responsibility for that portion of increased capacity which is directly related to its usage of the facilities. This method will permit adjustment in characteristics of the rate class and its imposed load on the system when overall demand undergoes a change. For each rate class, the increase in capacity which results is computed, and the class bears responsibility for the extra costs incurred in providing service at the peak period.

The model being described here is structured so that the incremental costs associated with providing sub-classes of service can be computed, and their interactions with all other services and sub-classes of service recognized. Existing nonlinearities can also be analyzed, through the total cost-total revenue methodology outlined; statistical experiments can be designed so that, by interactive runs of the model, the additional costs associated with various service classes and sub-classes can be estimated and their interactions determined. Employing this procedure has the disadvantage of necessitating additional computer runs. However, through the use of computer technology and good statistical experimental design, the process can be accomplished with reasonable effort. The insight to be gained is sufficient justification for the approach, for it provides meaningful estimates that incorporate the major interactions and are not based on arbitrary allocation schemes.

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[7] See Reference [4]

7. The model: a modular approach

The analysis which was developed to meet all of the objectives and requirements given in the foregoing sections can best be described as a research facility which utilizes analytical modeling techniques. This facility was developed as a system which could evolve over time. A modular approach was adopted, in order that various components of this system could be studied apart from the rest of the structure, with an eye to refining or extending them, or determining appropriate levels of aggregation. In this way, too, each component could be made to reflect changing requirements and conditions without necessitating changes to the entire model. In the analysis of such a complex process, an evolutionary approach was viewed as the most practical of the strategies which were considered.

The methodology. Detailed analyses are employed to simulate certain aspects of the demand-cost interaction, where the response is highly non-linear. One example of this is seen in the estimation of capital investments associated with peak demand requirements, that is, the analysis which estimates investment changes associated with peak period equipment requirements. Certain selective simulation procedures, coupled with statistical estimation techniques, were used to study the major operating characteristics of the supply system. The overall analysis is disaggregate, by geography as well as by plant component. The objective of this approach was not principally to develop precise estimates for each level of aggregation within the supply-cost structure (individual circuit groups, for example, or switching machines), but rather to develop more reliable total estimates of the incremental costs resulting from structured demand changes by allowing exogenous data to be employed and various interactions to be modeled and studied which would not be compatible with a less detailed approach. Since one of the major objectives which was stated earlier was to build a research facility which would allow examination of other modeling methodologies, an aggregate approach to the analysis was regarded as inadequate.

The model is, however, structured to deal with the demand-supply process as a whole, as an overall system, rather than with particular aspects. Trying to deal with one service or class of service or with some of the interrelated plant components apart from others sharing the supply network would be like trying to determine which part of a body's food intake provides the energy to move one finger.

Within the various modules, statistical averaging was used where response is smooth, where detailed information was not available, where definition of procedures or standards was lacking, or where analytical and computing capabilities were limited. The data bases and statistical parameters are structured to be replaceable, which allows flexibility in both the development of the analysis and its refinement. This structure permits study of the sensitivity of the model to changes in the major parameters, and will help provide a means for assessing the reliability and uncertainty associated with the estimates.

The cost philosophy used for the model was based on the concept of total incremental costs associated with incremental units of service supplied (see Section 4 p. 7). The

relationships of price and cost interaction to the major elements of plant, to peak period characteristics, to the utilization of plant, to the interactions among plant components, and to the interactions among network services and categories of service can be investigated through this model. The key strategy followed was to identify modular components that would interact with respect to their inputs and outputs, but that could be modified and developed independently.

Model processes. The next four figures show the model processes in various levels of detail. Figure 26 shows the three basic areas comprising the model's 18 modules, and the outputs and inputs of each. The three areas are:

(1) *Demand sector.* This sector, in which there are three modules, produces the demand estimates required for production cost considerations and translates them into forms suitable for the various components of the network supply system. Economic and price variables are input to market and revenue analyses, which forecast demand and revenue changes for the various categories of MTS and WATS service. Note that these demand and revenue changes may be represented as either positive or negative increments. The translation module processes the usage estimates, structuring them as input to the production sector modules. This involves primarily converting estimated billed messages and message minutes to offered load to the various relevant categories of the network supply system.

(2) *Production sector.* Each module within the production sector provides estimates of changes in facility requirements, equipment needs to satisfy these requirements, or investment or operating costs associated with the equipment needs. There are a total of 12 modules in this sector, of which the outputs are the total changes in book costs (associated with changes in capital investments) and the total changes in operating expenses associated with changes in demand.

(3) *Annualization and management sector.* In the final sector, costs and revenues are annualized. The results are then ready for analysis. It is then possible — and highly probable — that the price variables will be altered and another "run" undertaken in order to obtain new results for comparison and further analysis.

A fourth sector contains a single module which analyzes the effects of separations procedures on the revised book costs and operating expenses, as well as on the forecast revenues. It is an optional procedure within the model, and in normal applications will be used together with the non-separations results in comparisons of various price change effects by management.

FIGURE 26

OVERVIEW OF THE MODEL

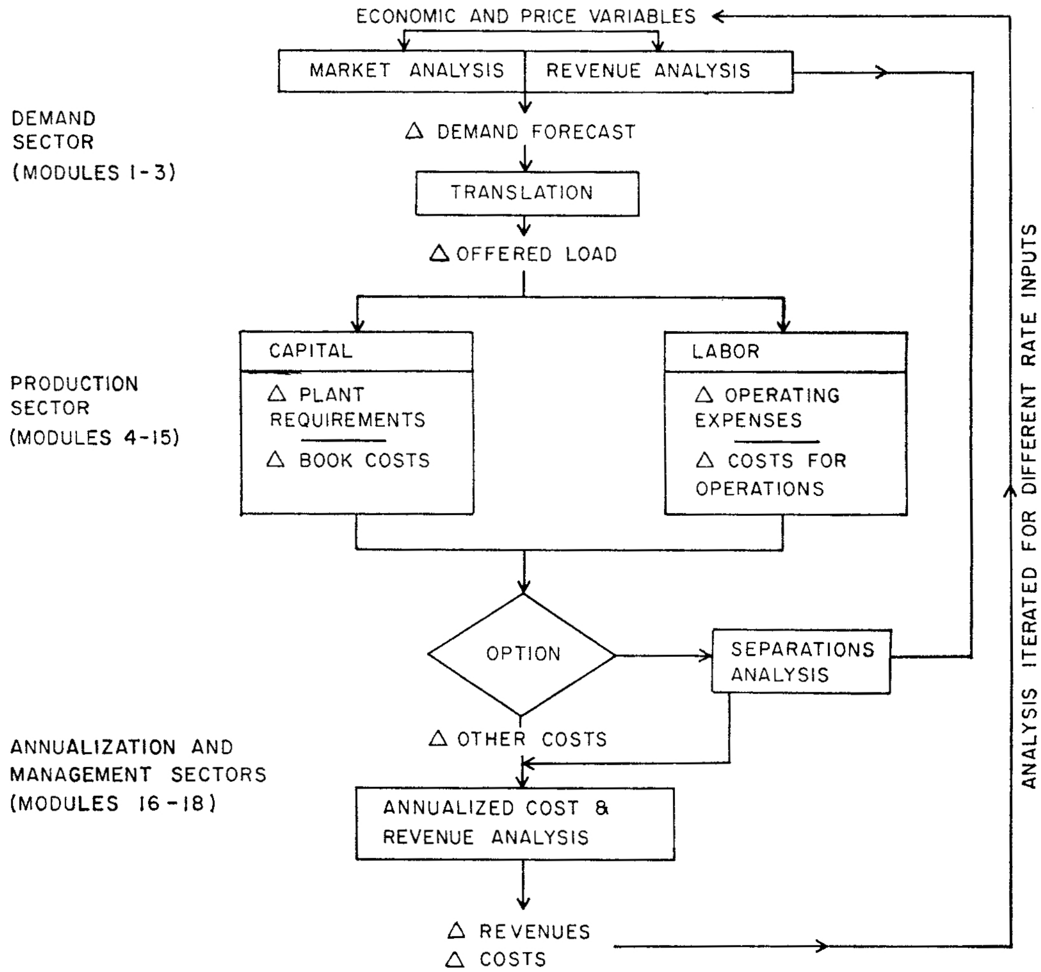


Figure 27 illustrates the demand sector. Various demand parameters are input to the two demand models currently in use, one for inter state MTS service and the other for interstate WATS. (As this is written, these interstate models are still being researched. Intrastate models, also being researched, are still not at a sufficient stage of development for inclusion in this demand-supply model.) Figure 27 also shows the translation process required to develop usage inputs in appropriate forms for the production modules.

FIGURE 27

INTERACTION OF DEMAND SECTOR OF MODEL WITH PRODUCTION AND ANNUALIZATION SECTORS

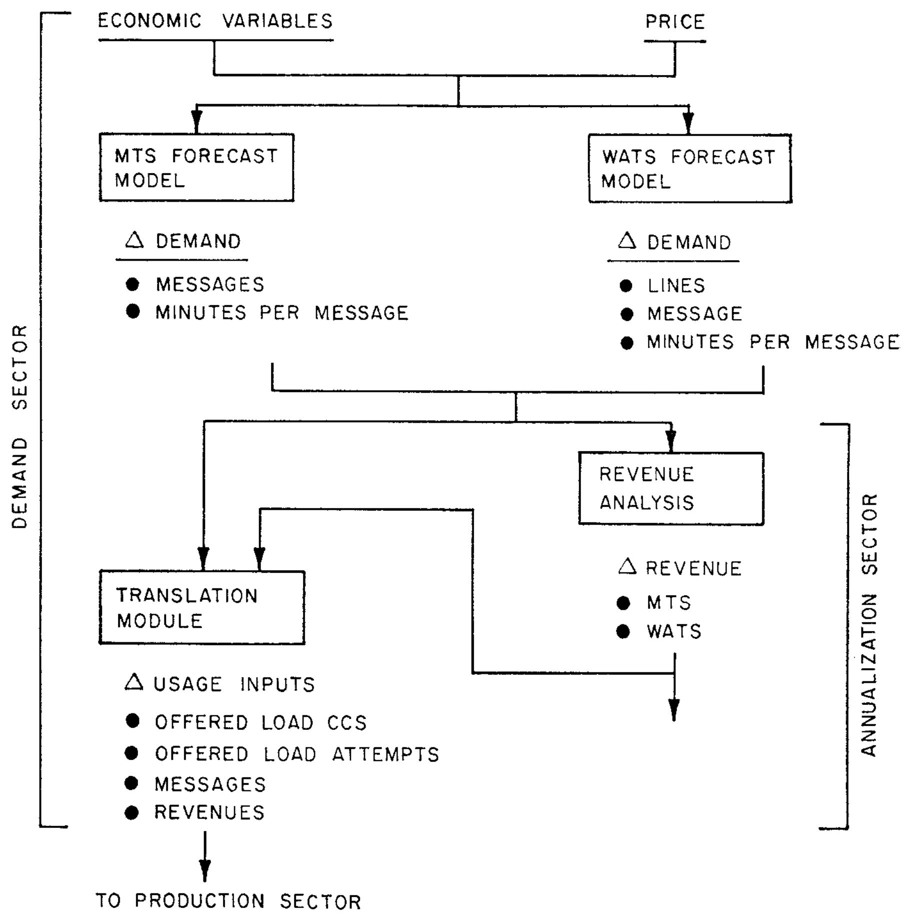
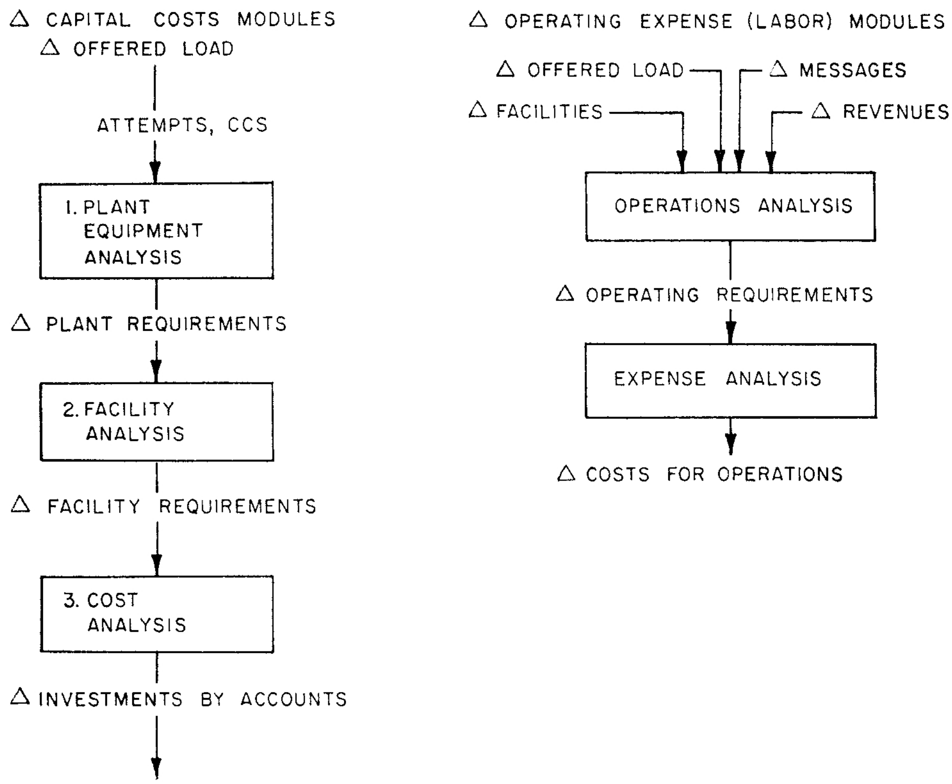


Figure 28 illustrates the production sector, in which the capital investments and operating expenses are estimated that might result from a change in interstate toll demand. The modules on the left in the diagram were identified to permit the basic characteristics of the telephone supply system to be studied and related to the price vectors. Those on the right in the diagram were structured to permit analysis of the various labor-sensitive costs — the operating expenses associated with providing service.

FIGURE 28

PRODUCTION MODULES



PLANT EQUIPMENT MODULES

CENTRAL OFFICE EQUIPMENT
- IX CIRCUITS
- TOLL CONNECTING CIRCUITS
- TOLL DIAL SWITCHING EQUIPMENT
- MANUAL SWITCHING EQUIPMENT
- SPECIAL SWITCHING EQUIPMENT

LOCAL EXCHANGE EQUIPMENT
LAND AND BUILDING
MATERIALS AND SUPPLIES
MOTOR VEHICLE AND WORK EQUIPMENT
FURNITURE AND OFFICE EQUIPMENT

OPERATING EXPENSE MODULES

TRAFFIC
COE SWITCHING EQUIPMENT MAINTENANCE AND REARRANGEMENT
COE CIRCUIT EQUIPMENT MAINTENANCE AND REARRANGEMENT
BUILDING MAINTENANCE
OSP MAINTENANCE
CIRCUIT TESTING
COMMERCIAL AND MARKETING
ACCOUNTING
GENERAL ADMINISTRATIVE EXPENSES
SOCIAL SECURITY, RELIEF, AND PENSIONS
OTHER OPERATING EXPENSES

The first module on the capital costs side (plant analysis) transforms change in offered load to peak capacity requirements. The following component in the diagram represents the process of transforming peak capacity requirements into facility requirements over the

planning period. This analysis recognizes the technology available for the planning period and the investment costs by facility categories, and estimates the incremental facilities that will be most likely to result in the minimum-cost investment program over the planning period.

The third component on the capital costs side is the cost analysis. Here, the incremental investment is computed by applying more detailed cost functions than those employed in the second step. Discounting principles are employed, and the output of this component comprises the incremental costs associated with the change in demand or offered load that is being evaluated.

The operating expense modules were developed to estimate operating costs based on requirements using rate and supply parameters. For example, the traffic expense module (which is more fully described in a later section) required a more detailed analysis than some of the other expense calculations, because of the non-linear aspects of the operator-scheduling problem. Certain other expense modules did not require a detailed level of analysis, and more conventional statistical estimating procedures were therefore employed.

Figure 29 shows the total module structure and logic flow of the analysis, including the module in which the various estimates are brought together to form the basis for developing revenue requirements associated with the particular changes in demand being studied. Within each of the detailed module descriptions, the variables employed are classified as exogenous, state (status), or endogenous. These classifications describe the relationship of data to particular modules of the analysis or to the overall structure and, although the usual definitions apply, some clarification is needed:

(1) *Exogenous variables.* These are input variables with respect to the analysis, and have been predetermined independently of the module being discussed. That is, they act upon that module but are not acted upon by it. The exogenous variables are partitioned into two classes, those which are exogenous to the total model and those that are exogenous to particular modules (having been produced as output by other modules).

(2) *Status variables.* These describe the state of the system or of one of its components at the start of a particular time period. They interact with both the exogenous and the endogenous variables, according to the assumed functional relationship of these within the system. The value of a status variable during a particular time period may depend on the value(s) of one or more of the exogenous variables during preceding periods and also on the value of certain output variables.

(3) *Endogenous variables.* Endogenous variables are the dependent or output variables of the module and also the system, and are generated from the interaction of the exogenous and status variables according to the system's operating characteristics. These variables are classified by modules, and for the overall model structure.

The functional relationships describing the interaction of these variables and of the components shown in the diagram (Figure 29) may be identities or descriptions of operating characteristics. Both are used to generate the behavior of the system under certain conditions.

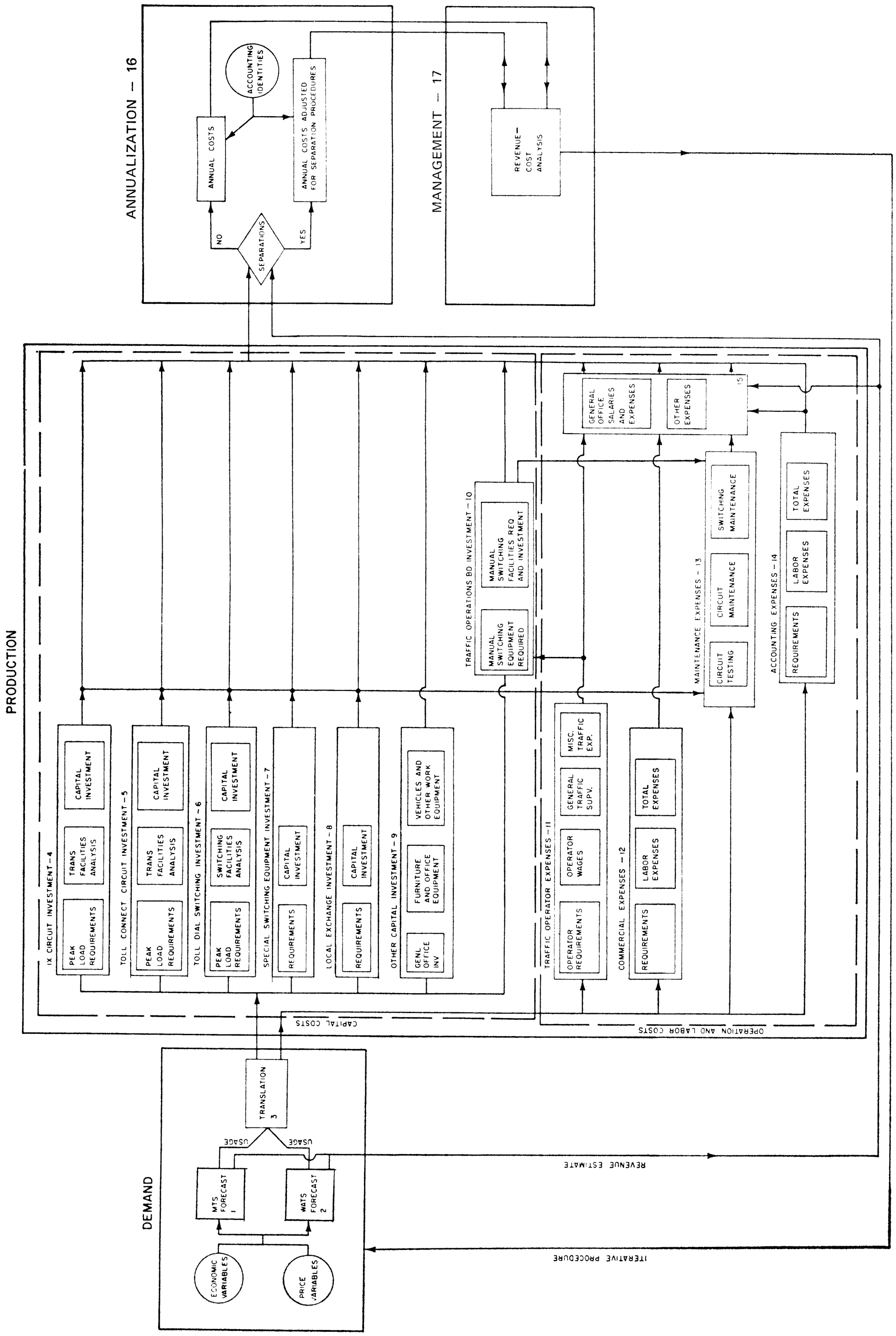


FIGURE 29
MODULAR STRUCTURE AND DETAILED LOGIC FLOW

Module 1: MTS demand. The model that estimates demand for interstate Message Telecommunications Service (MTS) was developed during April 1966–July 1967 in the Long Lines Department of AT&T, under the leadership of R. Auray, Director of Business Research. The market for which message telephone service revenues and traffic effects are predicted is the interstate domestic area comprising 48 contiguous states and the District of Columbia. (U.S. message traffic to and from Hawaii, Alaska, Canada, Mexico, and overseas countries is not considered.)

Data. The volume of business during a single month (October) is simulated in considerable detail by this model, which is based on MTS October data accumulated over many years. The variables are:

(1) Exogenous variables

  (a) The model inputs are a set of rate plans (prices) which would cause shifts in usage or in demand for particular classes of service, and might stimulate new demand. The model also uses data pertaining to "substitute" services (private line, WATS), to which demand might shift if the rates for MTS were to change.

  (b) Variables which represent economic fluctuations, such as rapid expansion or contraction of income, are used to show the effects on demand for interstate message telephone service. These variables are disposable personal income (DPI) obtained from National Income Accounts, and AT&T's index of industrial activity. Both are measured in terms of ratio to a trend line fitted to post-World War II data, with DPI being first converted to a constant dollar basis.

  (c) Another input variable developed from analysis and measurement of basic growth is the rate of growth which would prevail in a sector of the market if other effects were held constant (such as economic growth trends, rates, substitute service demand growth).

  (d) Finally, some minor variables are included. These include a measure of the variation in the number of effective business or trading days during a given study period; for example, October may have anywhere from 21 business days, 5 Saturdays, and 5 Sundays to as many as 23 business days, 4 Saturdays, and 4 Sundays, and the addition of two weekdays can cause a perceptible increase in message volume. Another minor variable reflects shocks, such as hurricanes, tornadoes, floods, international crises, stock market fluctuations, and so on, which cause surges in telephone demand. A third minor variable represents the impact of domestic military conditions upon message telephone demand; this variable is used to differentiate between the effects of the military manpower buildup characteristic of wartime periods and the effects of the concurrent surges in economic activity. (This variable is primarily of historical use, in analyzing past demand, rather than of present significance, since the effects of the military variable on the present demand for message telephone service are small.)

FIGURE 30

SECTORS OF THE MTS MARKET ANALYZED BY THE MODEL

	NUMBER
MAJOR SECTORS:	
CUSTOMER CLASS	3
LENGTH OF HAUL	15
TYPE OF CALL	2
TYPE OF DAY	3
TIME OF DAY	25
	6,750 SECTORS
ALLOCATION OF EACH SECTOR'S MESSAGES BY:	
LENGTH OF CONVERSATION	90 (APPROX.)
TOTAL SECTORS	600,000 (APPROX.)

(2) Endogenous variables

The variables generated by this model represent the specific reactions of many individual sectors of the market to changes in the causative (exogenous) variables. Figure 30 shows the market subdivisions reflected in this simulation; there are 6,750 separate sectors, or "cells." These are produced by dividing the market into customer classes (business, residential, and coin), then into 15 lengths of haul, then into two types of call (person-to-person, station-to-station) and further into two types of station traffic (operator-assisted and DDD), then into three types of day (weekday, Saturday, Sunday), and finally into 25 times of day (23 one-hour periods and two half-hour periods). For example, one of the 6,750 sectors of the market is the residential demand for service during the 8-9 a.m. period on a weekday, for station to-station DDD calls from 1 to 8 miles in length. Each such sector of the market has an explicit estimating equation within the model, called a message generator equation.

The model. The model estimates revenues and the lengths of the calls within each of the above sectors. These are estimated by distributing the messages within each cell by empirically derived lengths of conversation functions, which are mathematical formulas representing the typical lengths of call for each sector (based on observed usage). This results in approximately 90 additional subdivisions of the market, as shown in Figure 30. The procedure is to distribute messages in each cell cumulatively by one-minute intervals, starting with 0 to 1 minute messages, then 1- to 2-minute messages, 2- to 3-minute messages, etc., until 99.9 percent of the messages in that cell have been distributed; the remaining 0.1 percent are allocated to the next higher minute above the last minute considered, and in the typical market sector it is necessary to extend out to about 90 minutes for full distribution of the messages — hence, 90 subdivisions.

To summarize, the endogenous variables are predicted message volumes (generated

individually) by sectors of the MTS market, and individual estimates of messages by length of conversation. Each of the individual message estimates has an associated unique price, because the rates for MTS are scaled by all of the categories of service and by length of conversation (charges depending upon the initial period, whose length may vary, and the overtime period). Therefore, revenue estimates are also produced for each of the sectors of output. Finally, message volumes and revenues are accumulated into a variety of summaries for subsequent analysis.

Message generation. The model is made up of a series of equations pertaining to individual sector of the market. The form of the equation used to generate messages for each of the 6,750 cells described under endogenous data is the following:

$$\hat{M}_i(c) = a_0(c)\left[X_{1,i}^{a_1(c)} X_{2,i}^{a_2(c)} X_{3,i}^{a_3(c)} X_{4,i}\right] X_{7,i} a_6(c)^{X_{6,i}} a_8(c)^{X_{8,i}} + X_{3,i}^{a_3(c)} \sum_{j=1}^{150} m_{c,j}$$

where

c	= cell number (1:6750),
i	= year,
$\hat{M}_i(c)$	= message volume in category c for year i,
$a_0(c)$	= constant representing the number of messages in the base year,
$a_1(c)$—$a_3(c)$	= estimated coefficients used to weight explanatory variables,
$a_6(c)$	= average growth rate, 1948–1960,
$a_8(c)$	= average growth rate, 1961–present time,
$X_{1,i}$	= military variable in year i,
$X_{2,i}$	= economic variable in year i,
$X_{3,i}$	= price variable in year i,
$X_{4,i}$	= competition and shocks variable in year i,
$X_{6,i}$	= dummy variable for years 1948-1960,
$X_{7,i}$	= equivalent business days variable in year i,
$X_{8,i}$	= dummy variable for years 1961-present time, and
$m_{c,j}$	= messages shifted from one hour to another (25), or from one class to another (station or person), or from one type of day to another (average business day, weekend day, or holiday).

All variables except a_6 and a_8 are indices, where 1960 is defined as 1.000. The model defines estimated messages in period i in terms of changes from the base period situation due to the effects of time, rate changes, economic conditions, and other influences. The split growth factors (a_6 and a_8) permit evaluation of noticeable changes in growth rate such as that which occurred between station and person traffic due to the rapid growth in DDD capability between 1958 and 1962.

The additive term

$$X_{3,i}^{a_3(c)} \sum_{j=1}^{150} m_{c,j}$$

in the message generator equation represents the cross-elasticity effects of price, where rates may cause usage to shift to other classifications of service. The summation over 150 cells

represents the fact that the model considers the possibility of shifting from potentially 150 other sectors of the market into a given sector (literally only 149, excluding the sector under consideration). The 150 sectors include shifts in three types of day groupings (from weekday to Saturday or to Sunday, or vice versa), two classes of traffic (person to station, or vice versa), and 25 times of day (from one time period to another). The model does not consider shifting of traffic from one mileage band to another due to a rate change, or from one class of customer to another.

Notice also in the equation that the summation

$$\sum_{j=1}^{150} m_{c,j}$$

which represents the aggregate messages gained by a given cell or sector of the market from all other cells in response to a rate change are then "stimulated" just as if they were originally included in this sector. The term

$$X_{3,i}^{a_3(c)}$$

outside of the summation sign indicates this stimulation, and indicates that when messages are shifted on balance from one sector of the market to another, the customers thus shifting will also generate additional calls (if there is a lower rate in the new cell) in the same proportion as those generated by customers who already were in that sector of the market. The process by which messages are shifted and reclassified was found to be dependent on several variables, which are:

(1) The reluctance of customers to shift from their given calling patterns to other calling patterns (particularly business customers).

(2) The desirability of calling at the time represented by cell C_1 versus the time represented by cell C; the more nearly equal the desirability of calling at the two times (defined as varying be tween 0 and 1), the more significant the shift due to a rate change.

(3) The time distance between the two cells, in number of hours which the caller has to wait in order to shift to the new cell and hence take advantage of lower rates.

(4) The absolute time required for the customer to wait in order to take advantage of the lower rates. This is introduced in order to take into account the fact that waiting seemed to mean different things to different classes of customer, other things remaining equal. For example, business customers would be less likely to wait in order to gain a rate advantage, whereas residential customers might be willing to change their calling patterns.

After the shift and reclassification model had been initially formulated, a further refinement was added in the form of a constraint on the number of messages which could be reclassified: only 75 to 80 percent of the messages (total) in any given cell can now shift to other cells in response to rate differentials. This is needed to reflect the high urgency nature of some calls, such that no matter what rate incentives are offered, the caller is unwilling to shift his usage to another time period.

The reclassification formula seems to fit observed effects which occurred in 1963 following the introduction of the lower rates after 9 p.m., and again in 1965 when the after 9 rates were first offered earlier in the evening (after 8). In addition, this form of the model has been a useful indicator of shift response actually observed in other subsequent rate changes, having correctly indicated that there would be little shifting or reclassification of customer usage in response to drastically lowered rates in the midnight-to-7 a.m. period.

Length of conversation distribution. The method whereby messages generated by the message generation portion of the model are distributed over various lengths of conversation for the purpose of providing a usage breakdown which could be related directly to revenue has been discussed briefly above.

A general mathematical expression was developed for approximating the family of curves which represent these distributions of calls. A log normal distribution was found to adequately approximate the actual distributions, and this type of distribution also had several additional advantages.

Given a log normal transformation, the actual length of conversation distributions were relatively easily and closely approximated. In addition, since the distribution function required only two parameters (mean and standard deviation), it was both simple and effective to extrapolate these distributions forward and backward in time. It appeared that, over time, the standard deviation of the distribution did not change, but the mean of the distribution did. Consequently, the extrapolations were performed holding standard deviation constant, which had the effect of maintaining the slopes while shifting the line to the right as a function of time. Good results were found in simulating prior or subsequent length of conversation distributions by this method.

Through this extrapolation, an additional dynamic effect was introduced into the model. By expressing the model's average length of conversation parameters as a function of time, the model is able to show a persistent change over time which corresponds to observable phenomena. Here, we have illustrated the steady but slow growth in average paid minutes. Excluding the time-change element from the model would have led to a persistent understatement of revenues, as the model projected into the future; the model would have underestimated the proportion of long conversation messages (and overstated the short).

Derivation of coefficients. Those familiar with statistical estimation procedures will readily understand the difficulties encountered in deriving the coefficients which have been described above. The alternatives were either to develop a simple, statistically tractable but less realistic and flexible model by omitting the necessary complex procedures for developing these variables or to persist, at the cost of statistical estimation difficulties, in order to obtain the realism and flexibility which were desired. The second course was decided upon, and a multi-step procedure was followed.

First, estimates of the economic, price, growth, base period messages, and military coefficients (the a's) used in the message generation equation were developed by conventional multiple regression techniques, applied to time series data for the period 1948-1965 inclusive, for the month of October. Then the aggregated data on October usage were analyzed so

that potential shifts and reclassifications resulting from rate differentials were encompassed by the input variables. For example, if a differential rate change occurred in one time of the day, the data for that time period were aggregated with other periods so that any shift of messages from one time to another was internal within the aggregation. The reason for this is to permit estimation of certain model coefficients by statistical regression techniques; unless the additive term (the shift and reclassification) is eliminated from the message generator equation, least squares regression techniques could not be applied. By aggregating the data as described whenever a rate differential occurred, the additive term is eliminated from the equation because it is effectively forced to 0.

The coefficients derived by least squares regression analysis of the time series data were then used as benchmarks in repeated simulations of other, disaggregated data, to estimate the shift and reclassification effects. In this step, the time series data which were aggregated in the first step were again separated into their disaggregate components and each element was then computed using the explanatory coefficients just derived. Deviations of the actual data from the computed values were taken as estimates of shift and reclassification.

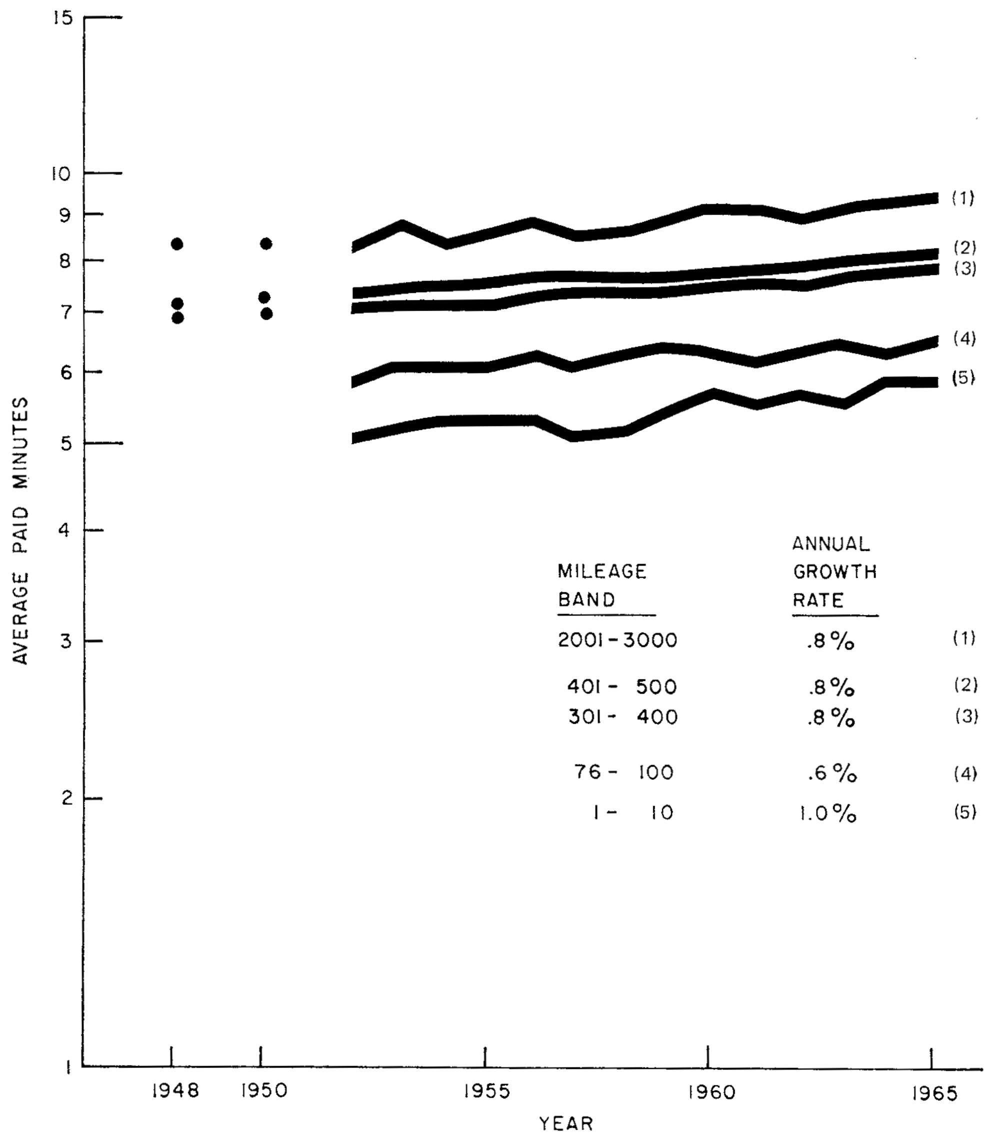
The initial regressions were done at high levels of aggregation (daytime vs. nighttime calls, station vs. person, etc.) to obtain estimates of total shifts and reclassifications. The data were then further disaggregated to permit analysis of the distribution of the shifts by successively smaller and smaller time periods. When reasonable and consistent distributions had finally been developed, repeated analyses were made using the shift-reclassification model to develop coefficients which would simulate the actual distributions of the shifts.

The coefficients used in the length-of-conversation mathematical formulation were developed by fitting lines to the actual length-of-conversation distributions. The required coefficients were the mean (average) and the standard deviation for the relevant length-of-conversation distribution for each sector of the market. Data were not available in sufficient detail to cover all sectors, but the mathematical functions were fitted to the smallest possible groups of sectors for which statistically reliable data were at hand. For example, an "average" distribution might have been developed for the category "residence — 221 to 292 miles — weekday — station-to-station — 5 a.m. to 6 p.m." This average would represent 13 sectors in the model (13 daytime time periods). The average coefficients thus derived were then assumed to apply to each of the time periods.

For the growth coefficients, trend curves were fitted to data such as those shown in Figure 31. These were then applied to the average length-of-conversation distributions.

FIGURE 31

LONG-TERM GROWTH IN AVERAGE PAID MINUTES (MTS)



The estimates of messages produced by the message-generator module were then distributed over an average of 90 length-of-conversation cells, resulting in a matrix of approximately 600,000 items. Revenue estimates Were then developed for each of these by the formula

$$\hat{R}_i = \sum_{c,l} \left[\hat{M}_{i,l}(c) P_{i,l}(c) \right]$$

where

$$\hat{R}_i = \text{estimated revenues for year i,}$$
$$\hat{M}_{i,l}(c) = \text{estimated messages in year } i \text{ for sector } c\text{, length of conversation } l\text{, and}$$
$$\hat{P}_{i,l}(c) = \text{rates for sector } c\text{, year } i\text{, length of conversation } l.$$ [8]

Summary remarks. Some summary indications about the values of the major coefficients thus derived might be of interest. The coefficients of the economic variable (a_2) ranged from close to zero to as high as 2.5, with low values being associated with calls having a short length of haul, and generally higher values being found as length of haul in creased. This suggests that long-haul (high average revenue per message) traffic was far more sensitive to economic fluctuations than short haul, low-cost traffic.

Price elasticity coefficients (a_3) range from about -.10 to the neighborhood of -.70. Here, too, the low negative values were typical for short-haul, low cost traffic with progressively larger negative values being found as length of haul (and cost of call) increased. Another generally consistent pattern was for price coefficients of "daytime" traffic to be noticeably lower (nearer 0) than "evening" coefficients.

With respect to the "pure growth" rates (a_6 and a_8), a wide range of results was obtained. With an average growth rate of about 10 percent per year, individual rates ranged from as low as significant *negative* growth (*e.g.*, -5 percent per year) to high *positive* growth (over 30 percent per year). Again, the tendency of these coefficients to increase as a function of length of haul was noticeable. As mentioned previously, a significant increase in the growth rate of station-to-station traffic after 1960 (a_8) compared with pre-1960 (a_6) was found, and a corresponding decrease after 1960 in growth of person-to-person traffic compared to pre-1960. Growth rates of business and residence sectors of the market were roughly equal, but both were persistently higher than public (coin) traffic growth, other things being equal. Finally, growth rates were generally higher for weekday daytime traffic than for evening and weekend traffic.

Coefficients (a_1) of the military conditions variable had a range (and a tendency to increase with length of haul) similar to the economic coefficients. However, the significance of this variable is minor; the range of the variable since the post-Korean War period has only been in the order of 1.00 to 1.03. Such variation has a minor effect on message results when compared with other variables. For example, the economic variable has ranged from 1.00 to 1.12; the price variable varied in a range of roughly .5O to 1.30; the growth ratios ranged from roughly .95 to 1.35.

Some persistent tendencies were also noted in the derived coefficients for the model's length-of-conversation distribution functions. Both the mean (average) length of conversation and the standard deviation in creased with length of haul; as a result, the coefficient of variation (the ratio of the standard deviation to the mean of each distribution) was roughly constant. The average length of conversation varied from less than 3 minutes at short hauls

[8] The actual period used in each year i for these estimates was the month of October.

to over 10 minutes at the longest mileages. Business and coin distributions tended to have lower average lengths of conversation than residence, other things being equal.

In the development of these coefficients, the availability of data had a considerable effect, as might be expected. As a consequence, judgment had to be exercised so that a logical system would be the result rather than a series of independent estimates which are unrelated to each other. For example, when a regression for one market sector produced an economic coefficient that was negative and a price coefficient that was positive, the implications were nonsensical (that an improvement in economic activity would result in lower demand and that a price increase would generate more demand), and were therefore ignored. Such aberrations could usually be traced to either very little variation in the independent variables or to a few outliers, and study of the confidence limits of these coefficients usually indicated that they were statistically insignificant. On the other hand, results for adjacent sectors of the market (either the next longer or shorter length of haul, or the next time period, etc.) were found to be more "reasonable" and far more often were statistically significant. Consequently, each set of coefficients (economic, price, and so on) was approached as a system rather than as a group of separate entities, even though they had to be developed for the most part independently.

Continuing studies are made of the need to update or revise the coefficients used in estimating MTS message demand. In addition, they are continually cross-checked against comparable results from other studies and approaches. This is one example of the evolutionary nature of the model.

MTS demand output. While the original implementation of this MTS demand model by the Long Lines Department was as an independent computer based forecasting tool, it was incorporated into the MTS WATS price-cost analysis with minor alterations. The original output, however, which comprised charts of the various endogenous variables for the usage categories described in the foregoing, was summarized into computer-readable tape formats, which were — together with the WATS demand forecasts to be described in the next section — then translated into the forms of input required for the determination of supply requirements, capital investment and labor expense, and revenue requirements.

Module 2: WATS demand. WATS service is offered, as stated in the introductory sections, in four basic combinations — full or measured time, and inward or outward. It is further subdivided into six service bands, as we have also described. A full-time band 1 outward WATS line from New Jersey would therefore permit a customer to place an unlimited number of telephone calls to the states falling within band 1 from New Jersey, for a fiat monthly charge. The WATS demand model that was formulated for the price-cost analysis of interstate toll telephone service is composed of three basic sections, or sub-models:

(a) A forecasting module,

(b) A price determination module, and

(c) A price response module.

In addition, there is a report-generation section which is operated as a separate component. The model was formulated so as to permit disaggregation, but economic data appropriate for this basis were not readily available at the time that the initial analysis was undertaken. However, other work employing exogenous variables is being investigated, with an eye to eventually developing a model which will yield the detail necessary for meeting the objective of providing management with the price-cost results needed for decisions.

Data. The following are the major variables employed in the analysis as presently structured, categorized as outlined in the introduction to the model description:

(1) *Exogenous variables*

The model uses WATS elasticity coefficients which were derived in external analyses of WATS price and cross-elasticities representing the interaction between WATS and MTS demand.

(2) *Status variables*

WATS customer usage data are available for a sample study period, given as individual WATS lines-in-use information, and a matrix of average customer usage.

(3) *Endogenous variables*

The model generates estimates of forecast WATS lines and of the revenues which would accrue, based on present rates for a specified period of time. It then generates average WATS customer prices, based on revised rates. A third category of variable is the estimated change in quantity of lines purchased as a result of the price change. A fourth category of output is the associated change in WATS revenues. Finally, revised rates for a forecast period are used to generate usage estimates.

Forecasting module. The first portion of the WATS demand model[9] estimates the expected growth that would occur based on the present set of rates. These expectations or forecasts are generated through the technique of exponential smoothing which was used because of (1) the need to forecast demand for 696 separate usage categories, (2) the short time constraint on model development and the desirability to get an operational model developed to use as a research facility, and (3) the capability offered by the method for adjusting swiftly and automatically to turning points in the data. (In the consideration of future modifications and refinements, other methods of analyzing time series may be appropriately coupled to the model.) The forecasting module developed used interstate outward WATS data only, due to the lack of sufficient inward data. However, the technique can be applied to inward data when available.[10]

Outward WATS service was introduced in 1961, and data on the number of lines in each of the six full-time and six measured-time bands for the fifty-eight WATS states were made available on a monthly basis from January 1962. Inward WATS service was introduced in

[9] Fitzgibbon, M. and Gitter, M. "WATS Demand Forecasting System," Management Sciences Division paper (AT&T Co.), August 26, 1971.

[10] Inward WATS service is a recent offering, and to date, a sufficient series of data does not exist for this analysis.

1969.

Three basic exponential smoothing models were sufficient to characterize the underlying processes for the period under consideration. They were constant, linear, and quadratic models. Table 6 summarizes the breakdown of forecast models that were developed.

TABLE 6

WATS DEMAND FORECASTING MODEL-OUTWARD

BREAKDOWN OF EQUATIONS BETWEEN CONSTANT, LINEAR, AND QUADRATIC AS OF MAY 1971.

NO. OF STATE-BANDS HAVING CONSTANT MODEL	241
NO. OF STATE-BANDS HAVING LINEAR GROWTH MODEL	443
NO. OF STATE-BANDS HAVING QUADRATIC GROWTH MODEL	12
TOTAL NO. OF STATE-BANDS	696

In the constant models, a smoothed statistic 51 is calculated for time t as follows:

$$S_t = \alpha X_t + (1-\alpha)S_{t-1},$$

where

$$\begin{aligned} X_t &= \text{ observation at time } t, \\ S_{t-1} &= \text{smoothed statistic at time } t-1, \text{ and} \\ \alpha &= \text{smoothing constant.} \end{aligned}$$

A forecast for T periods ahead, $\hat{X}_{t+T}$, is therefore

$$\hat{X}_{t+T} = S_t.$$

When the underlying process has a linear trend, two smoothed statistics must be estimated, S_t and $S_t^{(2)}$. They are calculated as follows:

$$S_t = \alpha X_t + (1-\alpha)S_{t-1},$$

and

$$S_t^{(2)} = \alpha X_t + (1-\alpha)S_{t-1}^{(2)}.$$

For a forecast T periods ahead, $\hat{X}_{t+T}$ is therefore

$$\hat{X}_{t+T} = \hat{a}_t + \hat{b}_t,$$

where

$$\hat{a}_t = 2S_t - S_t^{(2)}$$

and

$$\hat{b}_t = \frac{\alpha}{1-\alpha}\Big\{S_t - S_t^{(2)}\Big\}.$$

A similar procedure is followed for models involving higher powers of T.

The smoothed statistics S_t for time T are functions of the smoothed statistics in the preceding period. To obtain initial estimates of the smoothed statistics (*i.e.*, $t = 0$), initial estimates of the coefficients of the underlying processes are necessary. These are derived using regression methods. The regression coefficients are then used to estimate the smoothed statistics. For example, in the linear model the initial smoothed statistics, S_0 and $S_0^{(2)}$, are estimated as follows:

$$S_0 = a_0 - \frac{1-\alpha}{\alpha} b_0,$$

$$S_0^{(2)} = a_0 - \frac{2(1-\alpha)}{\alpha} b_0,$$

where a_0 and b_0 are the regression coefficients.

The choice of the smoothing constant α is critical to the success of the forecasting system, since it determines how rapidly the system adjusts to new data. The choice involves a trade-off between stability in the system on the one hand and ability to monitor and adapt to changes in the underlying pattern on the other; a relatively high value of α will give a relatively high weight to the most recent data and will quickly pick up changes in the trend but only at the expense of a decreasing ability to smooth out random fluctuations.

Adaptive smoothing is an attempt to deal with this problem by making α a variable, not a constant. By making α a function of the forecast errors, it is possible to give greater emphasis to the most recent data at times when the forecasts are poor (and hence the errors are large), while allowing low values of a when things are going well.

One way to achieve this is to monitor errors by means of a tracking signal and to set α equal to the value of this signal. Trigg and Leach[11] have proposed a tracking signal suitable for this purpose. It is defined as follows:

$$\text{Tracking Signal}(t) = \frac{\text{Smothed Error}(t)}{\text{Smoothed Absolute Error}(t)},$$

where

$$\text{Smoothed Error}(t) = \gamma\epsilon(t) + (1-\gamma),$$

$$\text{Smoothed Absolute Error}(t) = \gamma|\epsilon(t)| + (1-\gamma),$$

where

$$|X_t - \hat{X}|_t = \text{forecast error}.$$

The smoothing constant α is then set equal to the absolute value of the tracking signal.

This system has a number of advantages. First of all, the value of the tracking signal will always lie between $+1$ and -1; if the forecast errors tend to fluctuate around zero, so

[11] See Trigg and Leach [8]

will the tracking signal, while if they display a definite and continuing positive or negative bias, the signal will tend towards plus or minus one.

Secondly, the tracking signal will reset itself automatically when the forecasting system gets back into control.

There is a distinct model for each of the 696 state-band measured full combinations. The system provides the capability of accepting the latest data and updating each of the 696 models. The system also provides an automatic adjustment for a change in the underlying process. The most likely occurrence is for a constant model to change to a growth model. This type of fundamental change is detected by the tracking signal and monitored internally over a number of months until a process change is reasonably certain. The change to the new process is then carried out automatically by re-initializing the smoothing coefficients.

Price-determination module. The prices of many communications services are not at all explicit in the rate plans, particularly if one considers the price of substitute services within the same context. Analysis was undertaken to derive and study prices that would be equivalent under the three rate sets under consideration — *i.e.*, MTS, full-time WATS, and measured-time WATS. Since usage is the common denominator for all three, it was used to determine equivalent prices.

The WATS usage study, conducted once each year by AT&T's Marketing Department, provided individual WATS customer usage data for a 14-day period which were utilized in deriving equivalent prices. These prices were computed by applying the price which each customer would pay for his usage under the three rate structures. This computation involved manipulation of approximately seven million messages, for the study data used. Therefore, in order to lessen processing time, an average customer matrix was developed which collapsed the matrix generated by studying individual customer usage into one of more manageable proportions. While this has restricted somewhat the flexibility which would otherwise be available in analyzing various kinds of rate structures, it greatly reduced computational complexities and permitted an on-line analysis. Provision was also made for off-line determination of prices, so that rate structures which did not fit into the framework of the average customer matrix could be studied separately.

The average customer matrix was created by subdividing each cell in the 58 by 12 matrix of WATS usage (58 WATS states by 6 bands by 2 service categories — measured, full) into 11 time intervals, 30 mileage steps, and two types of day categories — weekends, weekdays. This yields a new matrix of 459,360 cells. Then the number of calls having the characteristics of each sub-cell was accumulated into those sub-cells. For those cells representing a range of conversation times, such as from 8 to 15 minutes, the actual average conversation times were computed and stored in the new matrix. The number of WATS lines in each state-band measured-full cell is also carried in the new matrix.

The information thus developed is felt to be adequate for computing MTS and WATS prices over a substantial set of alternative pricing schemes. The average customer matrix is accessed each time a new rate schedule is proposed for analysis, to estimate the effects of the price change. Prices and changes in prices are computed in two steps. First, the

base (*e.g.*, existing) WATS and MTS rates are run against the average customer matrix in order to generate what are called base effects. Now, the new or proposed rate schedule(s) is run against the matrix, producing revised effects. Incremental effects or changes are then a simple matter to obtain, by computing differences.

Three prices are used, and are defined as

P_{Tij} = the average customer MTS price for the ijth cell in the state-band matrix(58 x12),

P_{Mij} = the average customer measured WATS price for the ijth cell of the state-band matrix, and

P_{Fij} = the average customer full-time WATS price for the ijth cell of the state-band matrix.

Both these prices and the changes in prices (ΔP) are calculated then in three steps. First, P_{Tij} and ΔP_{Tij} are computed by applying the appropriate MTS rate (revised or base) to the number of calls in each usage cell, determining the charges that would have been incurred if all calls in each cell were priced out at that particular rate. This computation requires both the length of conversation and the length of haul parameters for each cell; the average customer matrix provides these data. The totals are computed for each cell, then divided by the number of lines in each cell to yield an average MTS price per line. The same method is used for determining both the base and the revised MTS comparable prices per line. The ΔP_{Tij} is simply the difference.

The second step, for computing P_{Mij} and ΔP_{Mij} is based on the total time for calls in each state-band measured-full cell in the average customer matrix. From the total time for individual customers, average overtime usage can be computed, depending upon the base number of hours in the measured WATS rate schedule being analyzed. From average overtime hours and the measured WATS rate schedule, the average customer measured WATS price, P_{Mij} can then be derived for both the base and the new rate schedules. ΔP_{Mij} is, once again, the difference between the two.

The full-time WATS prices, P_{Fij} and ΔP_{Fij} are taken directly from the full-time rate schedule, since the price is the same for any level of usage.

Price-response module. All of these prices are input to the price response formulation along with forecasts from the forecasting module and price and cross-price elasticities which are derived either from separate elasticity analyses or from direct inputs. The direct elasticity inputs can be useful in the analysis of the "what if?" questions which arise relative to the elasticity effect on quantities and revenues. All of this information, which is still being studied and developed, is exogenous input to the model at the present time.

The function of the price-response module is to perform the basic calculations from elementary demand theory which will yield the change in quantity of lines purchased at a given point in time as a result of a price change or changes. If Q_W denotes the number of lines forecasted for some given time; P_W, the price up to that time; ΔP_W, the proposed change in price; and ϵ, the price elasticity variable; then the change in number of lines due

to a WATS price change, ΔQ_W, is

$$\Delta Q_W = \epsilon \Delta P_W \frac{Q_W}{P_W}.$$

If there is a change in the price of MTS service, but no change in the price of WATS, and if P_T denotes the price of the MTS service with ϵ_c the cross-price elasticity, then the change in WATS lines due to the change in MTS price is

$$\Delta Q_W = \epsilon_c \Delta P_T \frac{Q_W}{P_T}$$

.

If there is a change in both WATS and MTS price, both calculations are made and the overall change in lines is the sum of the two effects. The change in the quantity of WATS lines purchased is carried out within each cell of the 58 by 12 matrix.

Reports generated. The forecasted number of lines for the time period under consideration for each state-band measured-full cell (Q) and the change in the number of lines (ΔQ) price change are generated as output for the revenue and incremental costs analysis portions of the overall demand-cost model. The WATS rate table is then used to compute revenue estimates under both the present and the new rate schedules, and the differences between the two. This is also done for each cell of the 58 by 12 matrix, for separate measured and full-time WATS totals and for a grand total. Other options are also open to the user of this analysis, with formats to be specified according to particular needs.

One final usage analysis is performed to derive estimates of the changes in usage which result when status information and price variables are updated or changed. The usage output gives information on the distribution of WATS calls by distance, conversation time, WATS states, bands, and category of service. The analysis consolidates this to produce two 15 by 18 arrays (15 aggregated rate mileage bands, 18 connect times), for both local and common time[12]. These arrays are produced based on the present rate structure, adjusted to reflect both the time difference from the base status variables and the change in usage due to a rate or price change. They can also be produced for the resultant usage in the forecast period, if desired (*i.e.*, for revised base plus change in usage).

All of these outputs are passed, together with the MTS demand model outputs, to the translation module where analyses are performed to develop appropriate inputs to the supply-cost modules.

Module 3: Translation. Usage translation is needed for three reasons: First, the demand usage outputs from the MTS and WATS forecast models are in the form of total system estimates by rate categories of completed messages and message minutes (that is, total system estimates by class of call, time of day, rate mileage, day of week, etc.), while the

[12] The rate mileage bands and time periods were defined in the description of the MTS demand model. The difference between local and common time is discussed in a later section, under Translation.

portions of the model that estimate supply costs require disaggregate inputs by major categories of supply facilities. This is because most facilities required to provide service are peak-load sensitive, (*i.e.*, engineered to give specific grades of service based on the load capacity during the peak periods). The telephone network is also designed to make efficient use of facilities by employing engineering procedures that allow for the most efficient utilization of equipment through recognition of the non-coincident traffic flows on circuit groups (paths between network "nodes[13]"). Therefore, the national usage estimates are translated into appropriate supply inputs by *major classes of facilities*, such as circuit plant, switching components, etc. These inputs must also be disaggregated on a geographical basis since intrastate toll prices, interstate and intrastate demand, and traffic characteristics vary geographically (see Section 5 pg. 15 for discussion). Disaggregate estimates are therefore needed to permit various price-cost relationships to be explored.

Another reason why it is necessary to translate the demand estimates is that the forecasts are in terms of completed messages and paid conversation minutes, while the supply system requirements are sensitive to the *total offered load* to the system including set-up time, operating time, attempts or incomplete calls, and non-charged calls. It is also important to recognize that the different equipment elements are sensitive to various combinations of the components of offered load, and changes in price can affect these components in many different ways.

Translation is also required because the supply system is an input output process dimensioned in both time and space. The demand estimates from the MTS model are aggregate geographical estimates based on local time (since all rate plans are a function of local time). The supply system is, as we have stated earlier, in many respects like a giant computer with many remote terminals operating in real time. The remote terminals (station and PBX equipment) are requesting service from the system all over the nation, and thus the supply system and its interrelated components are handling a large number of re quests simultaneously, efficiently utilizing resources by passing calls back and forth over various routing patterns to take advantage of idle components.

Translation of the demand model outputs is diagrammed in Figure 32 (by rate-sensitive classes). Essentially, the messages and message minutes in which predicted demand is expressed are converted to suitable offered load to the supply system (on a common time basis) in the units required for determination of incremental facilities and costs (see summary of outputs).

Data. The following variables are required in this module:

(1) *Exogenous variables*

The following are the categories of exogenous variables used in the translation phase:

(a) MTS demand model predictions by: type of service separately (*i.e.*, interstate vs. intrastate); times of day (hourly periods associated with rate classifications); rate

[13] A node is a location in the network which contains switching equipment connecting one circuit group to another.

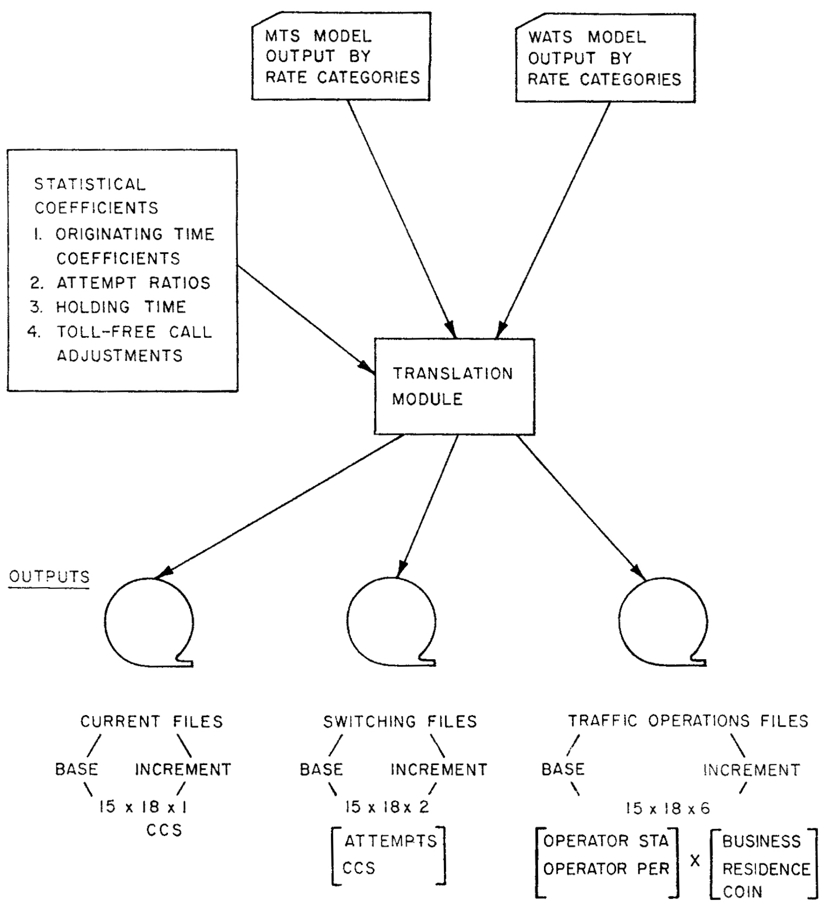
mileage bands; types of customer (business, residence, coin); 3 classes of call (DDD, person, station); and 3 types of day (weekday, Saturday, Sunday). This demand is expressed in annualized and average business days predicted by message volumes, distribution of conversation minutes, and revenues for a given year.

(b) WATS demand model predictions, by outward and inward services separately, in message volumes and message minutes by hourly periods, rate mileage bands, classes of customer (full, measured), and the six WATS areas as described earlier.

(c) Statistical operators or coefficients that are used to distribute the estimated change in traffic volume which would originate in each of the four time zones (Eastern, Mountain, Central, and Pacific) for each of the hourly periods of the day. Estimates are computed so that a volume of calls can be categorized by common type: operator-assisted station or person, direct distance dialed (DDD), or collect.

FIGURE 32

SUPPLY TRANSLATION PROGRAMS, SYSTEM FLOW DIAGRAM



(d) Statistical coefficients that are used to estimate attempts to-completed-calls ratios which are, in turn, needed for estimating total offered load.

(e) Statistical coefficients that are used to adjust billed conversation minutes and attempts to estimates of total offered load by equipment categories.

(f) Statistical coefficients that are used to adjust for toll-free calls, such as information calls, so as to estimate total offered load.

(2) *Endogenous variables*

The outputs produced by this module are in arrays of 15 by 18 elements — 15 rate mileage bands, 18 common time periods.[14]

These comprise:

(a) Total offered load by major service categories associated with toll circuit facilities for intertoll circuit facilities and for toll connecting trunking facilities. This does not include holding time associated with switching equipment. The offered load is expressed in CCS for a base load and an incremental load.

(b) Total offered load to switching equipment by major service category. (A major service category is equivalent to (1) interstate MTS, (2) intrastate MTS, (3) interstate outward WATS, (4) interstate inward WATS, (5) other.) This load is expressed in attempts and CCS, and consists of the following elements (once again in 15 by 18 matrices, by common time period and rate mileage bands):

1) Operator-handled attempts,
2) DDD attempts,
3) Total holding time for operator-handled switched CCS, and
4) Total holding time for DDD switched CCS.

These outputs are also produced for a base load and an incremental load.

(c) Total calls and attempts handled by operators by major service category. This load is expressed in the total offered load of calls that operators are required to handle to complete the calls placed by toll customers. The output is expressed for a base and an incremental load, and consists of 15 by 18 matrices for six categories of usage:

1) Business operator-assisted person calls,
2) Residence operator-assisted person calls,
3) Coin operator-assisted person calls,
4) Business operator-assisted station calls,
5) Residence operator-assisted station calls, and
6) Coin operator-assisted station calls.

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[14] Common Time is a representation of time periods across the nation on a single time scale so that total load at a given point in time can be analyzed.

The model. The base and incremental traffic files of data outlined above are developed separately by the translation analysis, in the following manner:

(1) *Adjustment to common time*

The network and equipment analyses must be conducted on a common time basis since the load is offered simultaneously to various equipment components. The demand model forecasts are based on local time, since rates or prices are structured on that basis. Therefore, the demand output must be adjusted from local time to common time.

To make this adjustment, prior analysis of data obtained through AT&T's Message Analysis Sampling Plan (MASP) was necessary. Statistical coefficients were developed based on estimations of the percentages of calls originating in the Eastern, Central, Mountain, and Pacific (Standard) time zones, by hourly periods, by type of customer (business, residence, public), by class of call (DDD, operator-assisted), and by rate mileage band. These coefficients were then applied to the MTS model forecasts to estimate the calls originating in those categories. It is assumed, in distributing the forecast calls for the system to the four time zones, that the new traffic predicted by the model (such as a 10 percent increase in the traffic in a particular cell of one of the 15 by 18 matrices) would be distributed in the same proportions as the previous time-zone pattern analyzed from the MASP base.

The MASP coefficients used for adjusting to common time were computed as follows. The base number of calls (obtained from the MASP sample) were developed and stored in an array X_{ijkl}, where $i =$ the 24 hourly periods of the day, $j = 15$ rate mileage bands, $k =$ the four time zones, and $l =$ the type of call (*e.g.*, operator-assisted and DDD). The percentage of the total traffic originating in each k was then computed, and an array of percentages (PC) for each i, j, l constructed.

Suppose that the MTS model predicts that at 2:00 p.m. local time, 400,000 calls are carried from 1 to 8 miles. Coefficients are applied to the 400,000 calls to spread them over the Eastern, Central, Mountain, and Pacific Standard time zones; then the hourly periods of the calls are adjusted accordingly. In the example under consideration, the estimations would be as follows: 46 per cent of the calls would originate in the Eastern Standard time zone, and remain at 2:00 p.m. Thirty percent would be allocated to the Central time zone and drop back to 1:00 p.m. Ten percent would fall in the Mountain time zone and be recorded at noon. And 14 percent would be allocated to the Pacific time zone, and be considered as part of the 11:00 a.m. load.

(2) *Estimates of incomplete calls (attempts)*

Each completed message can be thought of as representing a certain number of attempts, since a telephone customer does not always complete his call on the first attempt. For example, a busy condition is sometimes encountered (the person called may already be using the telephone); no one may be at home at the called party location; or an equipment busy condition may be encountered. Attempts-to-completion ratios were therefore developed for use in estimating incomplete calls. These attempt ratios are of

the general form

$$AR_{ijk} = \frac{NA_{ijk}}{NC_{ijk}}$$

where

$$\begin{aligned} AR &= \text{attempt ratio,} \\ NA &= \text{number of attempts,} \\ NC &= \text{number of completed calls,} \\ i &= \text{hourly period,} \\ j &= \text{day of week (weekday, Saturday, Sunday), and} \\ k &= \text{class of traffic.} \end{aligned}$$

Attempt ratios were found to vary over times of day, days of the week, and classes of traffic (DDD, operator-station, and operator-person). Distance and interstate vs. intrastate exhibited little or no significant effect in the statistical analysis conducted.

Table 7 is a breakdown of the statistical estimates of attempts to-completion ratios, by class of traffic, and by hour of day. The following can be observed among these estimates:

(a) Evening hours for weekdays have a lower percent completed calls than do daytime hours.

(b) Weekend attempts have a lower percent of completion on the whole than weekdays.

(c) Volume of blocked traffic has little effect on overall completion ratio.

TABLE 7

PERCENT COMPLETED CALLS PER ATTEMPTS, BY TIME OF DAY AND CLASS OF SERVICE*

HOUR**	DDD†	OPERATOR/PERSON	OPERATOR/STATION
WEEKDAY: 7-8AM	.746 ± .055	.474 ±.081	.795 ±.046
8-9	.719 ± .020	.594 ±.038	.842 ±.021
9-10	.688 ± .013	.558 ±.035	.854 ±.018
10-11	.671 ± .012	.626 ±.032	.813 ±.019
11-NOON	.632 ± .012	.570 ±.034	.810 ±.020
N-1PM	.648 ± .015	.308 ±.074	.789 ±.042
1-2	.647 ± .016	.547 ±.033	.858 ±.019
2-3	.689 ± .011	.614 ±.031	.841 ±.018
3-4	.680 ± .011	.536 ±.031	.825 ±.019
4-5	.661 ± .012	.583 ±.039	.804 ±.023
5-6	.635 ± .017	.600 ±.063	.741 ±.034
6-7	.612 ± .019	.570 ±.048	.747 ±.023
7-8	.571 ± .016	.520 ±.051	.697 ±.024
8-9	.502 ± .018	.395 ±.053	.756 ±.023
9-10	.543 ± .019	.405 ±.081	.778 ±.031
SATURDAY (AVG.)††	.598 ± .013	.444 ±.041	.741 ±.021
SUNDAY (AVG.)	.569 ± .016	.389 ±.044	.688 ±.023

NOTES: * ±1 STANDARD ERROR. STANDARD ERRORS ON PROPORTIONS WERE COMPUTED USING THE NORMAL APPROXIMATION TO THE BINOMIAL, e.g., STANDARD ERROR = $\sqrt{p(1-p)}/\sqrt{n}$

** INSUFFICIENT DATA AVAILABLE FOR 10PM - 7AM.

† AVERAGE FIGURE FOR WEEKEND CALLS BECAUSE OF SMALL SAMPLE SIZE DURING TWO DAY PERIOD

†† FOR DIRECT DIALED ATTEMPTS, PERCENTS SHOWN ARE BASED ON BILLED RATHER THAN COMPLETED CALLS, TO EXCLUDE CALLS DIALED TO TOLL-FREE INFORMATION (WHICH ARE NOT INCLUDED IN INPUT DATA TO THE MODEL)

The variations may be attributable to the difference in traffic patterns exhibited by business and residence customers. However, the data available for analysis did not allow this factor to be investigated.

The attempt ratios were applied to the forecast messages to estimate incompleted calls and to derive total attempts.

(3) *Estimates of additional calls* The demand forecasts do not include estimates of the traffic associated with toll-free information usage. This category of traffic is, however, highly correlated with completed toll usage. Analysis therefore was undertaken to develop coefficients which could be used to estimate both the relationships between the number of toll-free information calls and completed messages and the conversation time associated with this toll-free traffic category. The number of long-distance information messages is given by

$$I_i = M_i\left(\frac{C_i}{B_I} - 1\right),$$

where

$$\begin{aligned} I_i &= \text{number of information messages in period } i, \\ M_i &= \text{number of billed messages for period } i, \\ C_i &= \text{percent completed calls, and} \\ B_i &= \text{percent billed.} \end{aligned}$$

Additional conversation time due to this traffic is

$$D_i = \frac{32}{60} I_i,$$

where

$$\begin{aligned} D_i &= \text{conversation holding time,} \\ I_i &= \text{number of information messages, and} \\ 32 \text{ seconds} &= \text{average time (determined from analysis of a sample)} \\ &\quad\ \text{of information calls.} \end{aligned}$$

(4) *Line-hold vs. operating time adjustment*

Since the total offered load is required for estimating supply facilities, the supply model needs estimates of the total length of time in which a call "uses" facilities — *i.e.*, the message minutes (MMs) predicted by the demand model and the incomplete attempts have to be converted to estimates of total holding time in CCS (hundred call seconds). This is done by adjusting the estimated complete messages and related message minutes, the estimates of attempts, and the estimated noncharged calls for the time required for "operating" — setting up and disconnecting the call or attempt. This time varies by the class of equipment, based on where it enters the train in setting up the call.

TABLE 8

OPERATING TIME COEFFICIENTS FOR USE IN ESTIMATING INTERTOLL CIRCUIT FACILITIES

WEEKDAY	DDD		OPERATOR PERSON		OPERATOR STATION *	
	COMPLETE	INCOMPLETE	COMPLETE	INCOMPLETE	COMPLETE	INCOMPLETE
7-8 AM	.450	.510	.721	1.881	.282	.706
8-9	.313	.527	1.108	1.235	.273	.803
9-10	.328	.562	1.317	1.220	.268	.554
10-11	.325	.517	1.249	1.000	.275	.581
11-NOON	.323	.527	1.193	1.230	.266	.700
NOON-1 PM	.325	.562	1.013	1.402	.291	.951
1-2	.320	.575	1.191	1.139	.257	.711
2-3	.313	.518	1.384	1.384	.271	.997
3-4	.320	.568	1.419	1.116	.264	.748
4-5	.320	.548	1.382	1.170	.268	.706
5-6	.328	.580	1.270	1.589	.275	.546
6-7	.348	.623	1.108	.835	.291	.682
7-8	.365	.577	.760	1.320	.306	.647
8-9	.377	.558	.731	1.189	.301	.620
9-10	.368	.590	.764	1.290	.288	.667

* VALUES FOR NON-COLLECT ATTEMPTS. ADD 0.183 FOR COLLECT STATION TRAFFIC ONLY.

The ratio of operating time to calls, attempts, and conversation length varies considerably among certain categories of traffic; this variation affects costs and therefore is an important parameter in the study of price-cost relationships. The events which make up total operating time are equipment operating time (by components); operator set-up time (if applicable); ringing time; time elapsed before called party answers or before calling party hangs up if there is no answer or the line is busy; and disconnect time (of line-holding equipment).

The following 36 categories were investigated for 15 times of day in a statistical analysis:

Day of Week (3)	×	Equipment Component (2)	×	Class of Call (3)	×	Completion Category (2)
Weekday		Circuit Facilities		DDD		Complete
Saturday		Switching Facilities		Operator-Station		Incomplete
Sunday				Operator-Person		

TABLE 9

OPERATING TIME COEFFICIENTS FOR USE IN ESTIMATING SWITCHING EQUIPMENT (MINUTES)

WEEKDAY	DDD		OPERATOR PERSON		OPERATOR STATION *	
	COMPLETE	INCOMPLETE	COMPLETE	INCOMPLETE	COMPLETE	INCOMPLETE
7-8 AM	.748	.808	2.105	2.482	1.198	1.763
8-9	.611	.825	2.105	2.270	1.175	1.850
9-10	.626	.860	2.295	2.300	1.173	1.795
10-11	.623	.815	2.093	2.180	1.160	1.655
11-NOON	.621	.825	2.110	2.390	1.185	1.833
NOON-1 PM	.623	.860	2.180	2.895	1.237	1.895
1-2	.618	.873	2.138	2.260	1.110	1.818
2-3	.611	.816	2.282	2.420	1.170	2.118
3-4	.618	.866	2.288	2.290	1.178	1.923
4-5	.618	.846	2.330	2.338	1.220	1.843
5-6	.626	.878	2.172	2.377	1.062	1.848
6-7	.646	.921	2.165	2.223	1.230	1.858
7-8	.663	.875	1.808	2.618	1.363	1.838
8-9	.675	.856	1.902	2.197	1.305	1.988
9-10	.666	.888	2.167	2.462	1.232	1.817

* VALUES FOR NON-COLLECT ATTEMPTS. ADD 0.183 FOR COLLECT STATION TRAFFIC ONLY.

Two basic sets of coefficients were developed and used in the analysis, one for switching components and the other for toll circuit facilities. Tables 8 and 9 show examples of these coefficients in an average business weekday.

Module 4: Intertoll circuit investment analysis. The 15 x 18 arrays of offered load by major switched service classifications derived in the translation module are input to the production sector of the model, where incremental facilities requirements and associated cost and expense estimates are derived within modules corresponding to the various plant, capital, and labor categories. The first of these estimates the effects of an incremental change in toll demand upon costs of the facilities interconnecting Bell System Class 4 and above toll centers. These facilities are the intertoll circuits.

The methodology employed is highly detailed, and is based on a selective simulation procedure for those portions which are highly non-linear, and on statistical imputation for portions in which the response is smooth, or where procedures were constrained by the unavailability of data or lack of definition. The model does not attempt to re-engineer the network, but it does attempt to reflect the basic network engineering economic principles described in Section 5, particularly with regard to network and scale economies and utilization of plant; to provide levels of disaggregation that reflect traffic, major plant components, and geographic differences; and to incorporate sufficient market and rate detail to permit price-cost relationships to be explored in depth.

The procedure is divided into three parts. Part 1 is an analysis evaluating the incremental toll circuit requirements resulting from the new demand which is sensitive to peak load traffic, Part 2 estimates the transmission facilities associated with the incremental requirements over a given planning period, and Part 3 estimates the capital investment costs associated with the program.

Incremental circuit requirements, Part 1 of the analysis. The following are the categories of variables employed in the first part:

(1) *Endogenous variables*

- (a) Incremental offered load. These data are the incremental traffic offered to circuit groups and to networks (as defined in Section 5) as a result of a change in the demand for service. They are incurred by time of day (hourly, 18); by rate mileage step (15); and by major service classification (4).[15]
- (b) Incremental number of circuits. The model determines the number of circuits which would be required in order to serve the incremental offered load (a).
- (c) Distributions of incremental circuits summarized by various categories (*e.g.*, by group size, length of circuit group, etc).
- (d) Changes in peak loads for networks and circuit groups summarized by various categories (*e.g.*, busy hour, size of change, etc).
- (e) Δ Utilization factors (capacity or utilization of circuits for various time periods).

  These factors signify the change in utilization of circuits by type (high usage, final) at various time periods — network peak period, circuit peak period, total day utilization, and specific hourly time periods.

(2) *System parameters*

- (a) Grade of service (as defined in Section 5).
- (b) Engineering schedules. These schedules define the criteria which the circuits are designed to satisfy, *i.e.*, economic CCS for high-usage groups, blocking probability (*e.g.*, .01 percent probability of not obtaining equipment for completing calls) for final groups.
- (c) An empirical curve relating the cost of adding a circuit on an alternative route to the cost of adding a circuit in a direct route as a function of distance. This curve is used by the model to determine the economical division of traffic between the direct and alternate routes.

[15] The four service classifications are interstate MTS, intrastate MTS, interstate outward WATS, and Other. In the latter are aggregated interstate inward WATS, intrastate WATS, and TWX, each of which individually accounts for a very small contribution. In addition, insufficient data made it advisable to group these services.

(3) Exogenous input

The following variables are developed in other parts of the overall model, and are thus only exogenous to the intertoll circuit facilities requirements portion:

(a) System incremental hourly offered loads to IX circuits, both updated state variable arrays and arrays of incremental demand based on a price change. The system incremental hourly offered loads to intertoll circuits, generated by the two demand models and transformed by the translation module to estimates of offered load to intertoll circuit groups on a common time basis, are used within this analysis to determine the new facilities requirements. The arrays are derived in the translation module by time of day (18), rate mileage step (15), and major service classification (4) for an average business day (ABD).

(4) Status variables

(a) Base inventory circuit file.

This file provides a "snapshot" — a description applicable to a given point in time — of each of the circuit groups included in the analysis (15,000).[16] This information was derived from a System-wide inventory, and contains the engineered level of each group, its geographical location (*i.e.*, originating and terminating points), its length (route mileage), and its type (engineered on a high usage or final basis).

(b) Network homing arrangements.

The homing arrangements associate each circuit group with its network. There are approximately 1500 intraregional and 45 regional networks within the 10 regional areas of the United States.

(c) Base circuit group hourly profiles.

This file contains hourly traffic profiles. The state profiles were developed from a study consisting of a census of completed messages for a 14-day period annually to provide a basis for separation procedures required by the regulatory bodies.

The hourly traffic offered to each circuit group was developed by routing (originating point to terminating point) the messages for the study period over their first-choice routes. The messages and message minutes for each circuit group were summarized by the following categories: by 15 rate mileage steps (that is, the airline distance used for billing or charging); by four service categories (MTS interstate, MTS intrastate, WATS, and other); and by 18 hourly time periods. The completed messages and message minutes were converted to estimated offered load (conversation length plus operating time, as described in Section 5), expressed as CCS:

[16] The analysis that will be discussed is designed to permit samples of these circuits and networks to be utilized in the analysis at the option of the user. The purpose of the samples is to simplify the analysis and reduce computing time. However, there are large differences associated with the networks, and the difficulty of analyzing network economies and geographical differences has required these options to be used with caution.

$$CCS_{ijl} = \frac{60}{100} MM_{ijl} + M_{ijl}\left[AR_{ij} OT_{ij}\right]$$

where

$$\begin{aligned} i &= \text{rate mileage step (15)}, \\ j &= \text{hourly period of the day (18)}, \\ l &= \text{circuit group (15,000)}, \\ MM_{ijl} &= \text{message minutes by mileage step, period, and group}, \\ M_{ijl} &= \text{messages by mileage step, period, and group}, \\ AR_{ij} &= \text{attempts ratio by mileage step and period, and} \\ OT_{ij} &= \text{operating time in CCS by mileage step and period.} \end{aligned}$$

The profiles represent the distributions of traffic that would be seen on each group during an ABD, which has been assumed for this study to be the controlling traffic load for engineering purposes.

(d) Base network hourly profiles.

This file is similar to the circuit group base inventory. It was developed using the circuit group information for the 1900 major networks associated with each final circuit group in the System. The matrices are by hourly periods (18), rate mileage steps (15) as described above, and major switched network service categories (4).

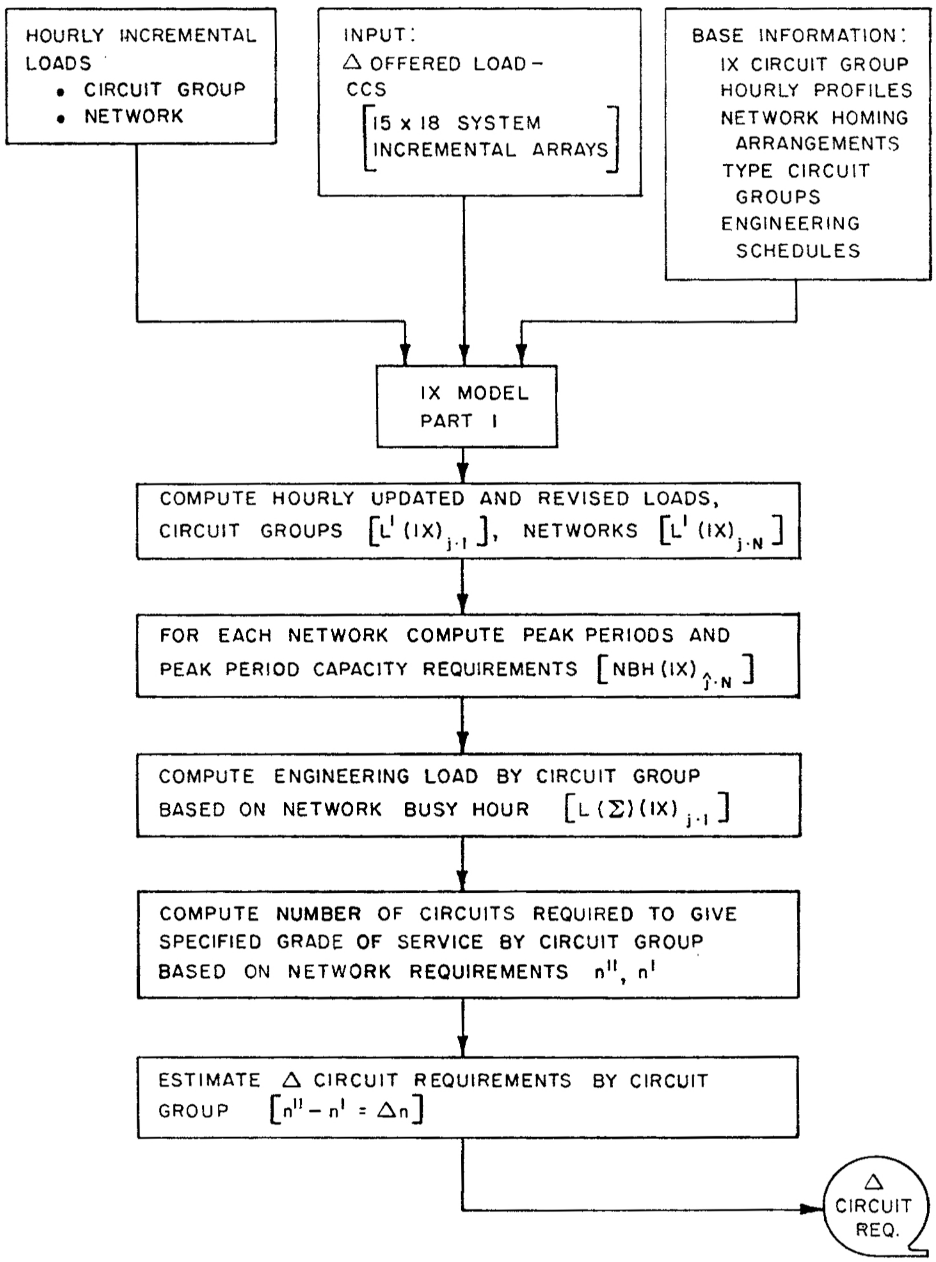
Model logic. Figure 33 is a simplified flow diagram which shows the basic logic of Part 1 of the intertoll circuit facilities model.

The procedure involves using the System-wide aggregate estimates of offered load developed for intertoll circuits in the demand translation modules in order to develop disaggregate estimates of Δ circuit requirements reflecting the geographical and network characteristics of the communications network. The expected change in hourly traffic for each of the categories of switched network service, by rate mileage steps, by major service classifications, and adjusted by the translators to a common time basis is used in this disaggregation process.

FIGURE 33

SIMPLIFIED MODULE LOGIC FLOW, PART I,

INTERTOLL CIRCUIT REQUIREMENTS ANALYSIS



The formulation to estimate the load increment for the lth circuit group (link) is

$$I(IX)_{ijsl} = L_{ijsl}\frac{\Delta B'_{ijs.} + B_{ijs.}}{B_{ijs.}} - L_{ijst},$$

where

$I(IX)_{ijsl}$ = the increment on link l for rate mileage step i, time period j, serv1ce s,

L_{ijsl} = the total base load in CCS, for the same categories of traffic (status profiles),

$\Delta B'_{ijs}$ = the projected System-wide CCS load (incremental) for proposed or revised rate structure, for the same categories of traffic (developed in the demand and translation phases), and

B_{iis} = CCS load associated with the rate structure in effect prior to the revision (also from the translation).

Note: B_{ijs*} updates the status profiles to current status, that is, adjusts the state variables for changes since the generation of study profiles.

Modules are being researched that will develop estimates of demand on a point-to-point basis, which will eliminate the need for this process. However, this procedure reflects the Δ demand changes in the rate mileage steps (15), times of day (18), and service categories (4) on circuit group profiles by assuming that the proportional change in traffic in each of the elements of the (15 x 18 x 4) arrays does not significantly vary in their distribution across the IX circuit groups in the system.

After the increments have been computed for a circuit group, they are summed over the 15 rate mileage steps (i) to give

$$PI(IX)_{.jsl} = \sum_{i=1}^{15}\left[I(IX)_{ijsl} + L(IX)_{ijsl}\right] \Big/ \sum_{i=1}^{15} L(IX)_{ijsl},$$

where

$PI(IX)_{.jsl}$ = the change in offered load in CCS for period j, services, on the lth link.

The change varies by circuit group, because of variations in the load profiles $\left[L(IX)_{ijsl}\right]$. These changes are then used to compute a vector of revised hourly load profiles for each circuit group:

$$L'(IX)_{.j.l} = \sum_{s=1}^{4}\left[L(IX)_{.jsl} * PI(IX)_{.jsl}\right]$$

where

$L'(IX)_{.j.l}$ = hourly revised load profiles for circuit group l, and time period j.

This process is first performed to update the status variables to reflect more current estimates of the circuit loads and is then repeated for a *revision or change* in circuit loads resulting from a new set of prices.

A similar procedure is employed to estimate updated and revised load profiles for the networks associated with each final circuit group,

$$L'(IX)_{.j.N},$$

where

$$N = \text{networks: } 1, 2, \ldots 1500 \textit{ intraregionals},$$
$$\text{and } 1, 2, \ldots 45 \textit{ regionals}.$$

These updated and revised network loads could also be derived by summing the updated and revised circuit group loads related to each network, but the computation is more time-consuming. The network busy hours are then calculated as

$$\hat{\jmath} = \text{value of } j \text{ which} = MAX_j L(IX)_{.j.N},$$

$$\hat{\jmath}' = \text{value of } j \text{ which} = MAX_j L'(IX)_{.j.N},$$

where

$L'(IX)_{.j.N}$ = the revised hourly CCS network loads, for network (N) and time of day (j), summed over all services and rate mileage steps, and

$L(IX)_{.j.N}$ = the base hourly CCS network loads, for network (N) and time of day (j), summed over all services and rate mileage steps.

The change in network load is

$$PNBHL(IX)_{...N} = \frac{L'(IX)_{.\hat{\jmath}'.N}}{L(IX)_{.\hat{\jmath}.N}},$$

where

$\hat{\jmath}'$ is the new network busy hour, and

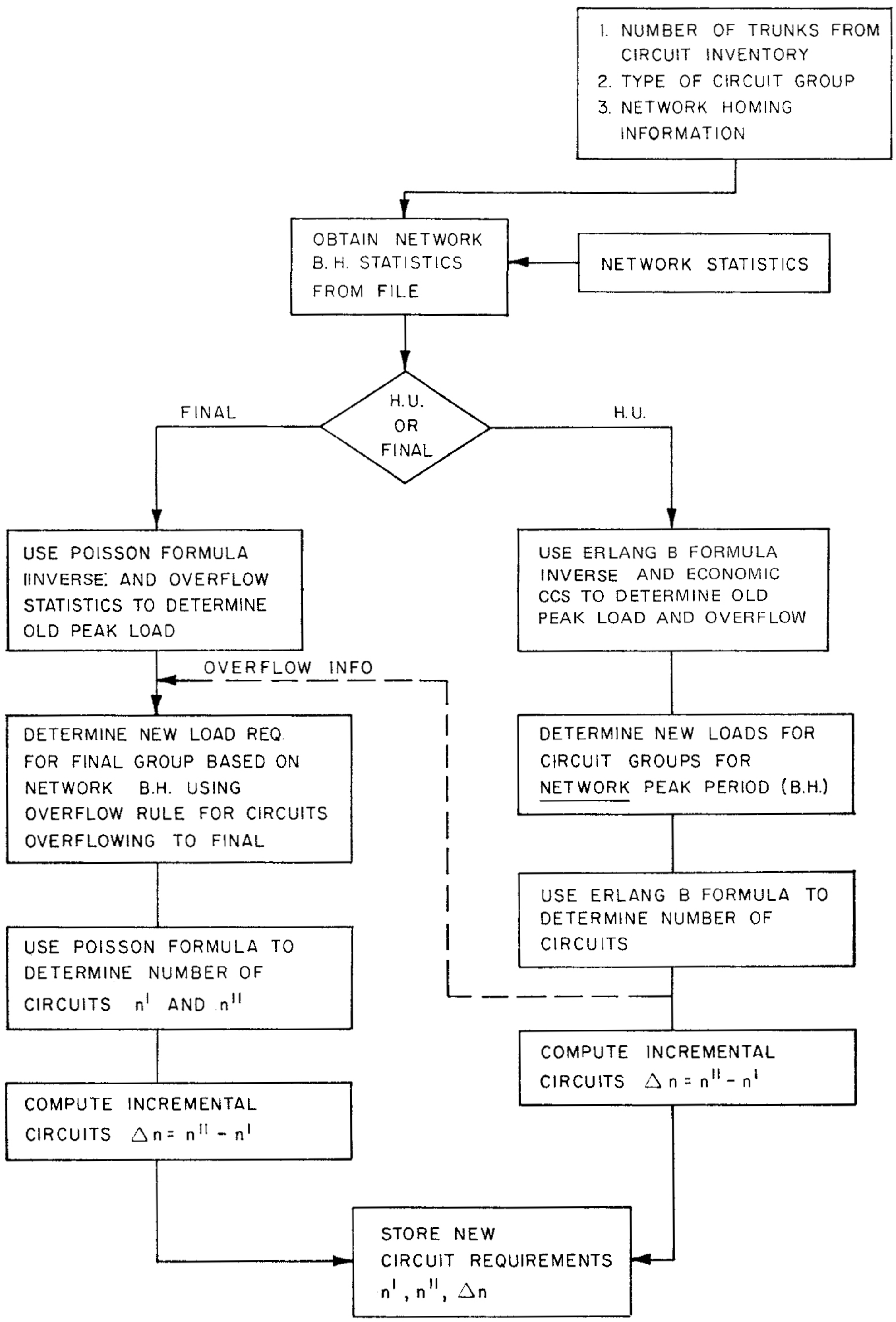
$\hat{\jmath}$ is the base network busy hour.

The next step in the analysis is to develop estimates of (a) updated state offered load variables, and (b) revised offered loads that are appropriate for estimating the incremental circuit requirements on the various circuit groups. Figure 34 illustrates this process.

The following assumptions were made with regard to estimating Δ circuits in the intertoll circuit analysis. The data used in actual practice as the basis for engineering circuit groups reflects the applicable busy season of the year for each group. The development of these profiles within the model would have been an impractical undertaking, and the status profiles derived for a study base period were therefore used instead. The analysis assumed that the primary difference, for studying price cost relationships, between the busy season and the sample study period is in level of traffic and not in distribution by time of day. The estimates of the ratio of post-incremental busy hour loads to pre-incremental loads for the study period are assumed to be invariant with regard to the busy season.

FIGURE 34

LOGIC FLOW DIAGRAM, DEVELOPMENT OF UPDATED AND REVISED OFFERED LOADS, INTERTOLL CIRCUIT FACILITIES MODEL



The revised or updated engineering loads can therefore be computed by first using the engineering schedule of each circuit group *in a reverse manner* to estimate the offered load which would have required the actual engineered number of circuits obtained from the census. The familiar Poisson blocking formula was used in *reverse* to estimate the state engineering load for final circuit groups, and the Erlang-B blocking formula was used *in reverse* to estimate the state engineering load and overflow corresponding to the alternate or high-usage groups. (See Section 5 for discussion of the engineering of alternate and final trunk groups.)

For high usage (HU) circuits, the Erlang-B blocking formula is based on an assumption of blocked calls cleared, *i.e.*, blocked calls will overflow either to other HU groups or to their final group and will not re-enter the system.

The basic formula used in estimating the state engineering load on the alternate or high usage route is

$$L(IX) = a\big[1 - B(n', a)\big],$$

where

$$B(n', a) = \frac{a^{n'} e^{-a}}{1 - p(n' + 1, a)},$$

$$\begin{aligned} p &= \text{probability of delay,} \\ a &= \text{average load submitted, and} \\ n' &= \text{number of circuits.} \end{aligned}$$

Therefore, estimates of the updated state load for each HU circuit group are developed in order to reflect the effects of a load change. The procedure utilizes network engineering theory to compute $L'(\epsilon)$, the revised load for each group, as follows:

$$L'(\epsilon)(IX)_{.\hat{\jmath}.l} = L(IX)_{.j.l} \left(\frac{(L'(IX)_{.\hat{\jmath}'.l}}{L(IX)_{.\hat{\jmath}.l}} \right),$$

where

$$\begin{aligned} L'(IX)_{.\hat{\jmath}.l} &= \text{revised load,} \\ L(IX)_{.j.l} &= \text{state offered load,} \\ \hat{\jmath} &= \text{base network peak period or busy hour,} \\ \hat{J}' &= \text{revised network peak periods,} \\ l &= \text{circuit group or link in a network.} \end{aligned}$$

Note: Since each circuit group is a member of two networks, one associated with each end of the circuit, and may be a member of an inter-regional network, the appropriate controlling network N is determined from maximum load conditions.

Once the estimate of the updated load is obtained, the above procedure is repeated to obtain estimates of changes in the updated load associated with price revisions:

$$L''(IX)_{.\hat{\jmath}.l} = L'(IX)_{.\hat{\jmath}'.l} \left(\frac{(L''(IX)_{.\hat{\jmath}'.l}}{L'(IX)_{.\hat{\jmath}'.l}} \right),$$

where

$$L''(IX)_{.\hat{\jmath}'.l} = \text{the revised load associated with price change, and}$$
$$L'(IX)_{.\hat{\jmath}'.l} = \text{the updated state engineering load.}$$

The next step in the analysis develops estimates of the number of HU circuits required to meet specified service criteria associated with the estimated loads for updating and price revisions. In estimating the number of HU circuits, it is necessary to estimate how much of the incremental traffic will be carried over direct (high-usage) routes and how much can be expected to overflow to final groups (*i.e.*, to reflect economies of scale resulting from alternate routing or funneling, non coincidence of traffic, etc.). The method employed by the model is based on the operating practice of seeking an optimum cost balance in the routing of the traffic between the alternate and direct routes. This requires a common denominator, which is the cost per busy hour CCS carried via each trunking path. The alternate route is more expensive than the direct, due to the longer distance and additional switching, but is larger and can therefore carry more traffic per path. The economic split of traffic between the two routes depends on the usage characteristics and the relative costs of the facilities and their call-carrying efficiencies.

The following is a brief description of the principles involved in HU circuit requirement estimation. The first fundamental principle is that an average cost ratio can be developed to represent an average annual incremental charge ρ for a circuit over the alternate route, including the cost of tandem switching, compared to the cost of a trunk over the direct route:

$$\rho = \frac{\text{cost of a tandem routing + switching costs}}{\text{cost of a direct routing}}$$

The greater the route miles, the lower the ratio of alternate route miles to direct route miles and therefore the lower the cost ratio, assuming that the last trunk in an alternate route will carry least load.

Then the load, $L(\gamma)$, that will be carried on the last trunk added to an existing alternate or tandem group must be estimated. The ratio of this load to ρ then represents the load, $L(\alpha)$, that should be carried by the last trunk in a direct group to result in the same cost per CCS as alternate routes:

$$L(\alpha) = \frac{L(\gamma)}{\rho}.$$

With this value and the number of CCS to be offered to the direct and alternate routes, the estimates of economic splits of traffic can be evaluated using appropriate engineering tables or algorithms.

To accomplish the above, the analysis employs an empirical function relating costs of adding a circuit on an alternate route to the cost of adding a circuit on a direct route as a function of distance. Empirical analysis has shown that the variable having the greatest influence on the cost ratio is the mileage in the direct route; switching and terminal facility costs can be assumed, for this analysis, to be rather constant, but line facility costs are a function of distance. The empirical formula used for determining the cost ratio is:

$$CR = 30/(10M^{\cdot 11}),$$

where

$$M = \text{mileage in the direct route.}$$

Circuits are then assumed to be added on the direct route until the cost per CCS carried on the last circuit added is at least equal to the cost per CCS carried on a circuit added to the alternate route (*i.e.*, costs are at the break-even point).

This procedure is employed to obtain estimates of

$$\eta' \cdots l_{HU} = \text{ circuits for update of state variables associated with } HU \text{ groups, and}$$
$$\eta'' \cdots l_{HU} = \text{circuits associated with price change.}$$

The incremental circuits requirements ($\Delta\eta$) are then computed for each circuit group and can be summarized by any or all of a number of classifications specified by the analyst, *e.g.*, geography, mileage classification, networks, etc., where

$$\Delta\eta \cdots l_{HU} = (\eta'' \cdots l_{HU} - \eta' \cdots l_{HU}).$$

The procedure to compute updated and revised estimates of incremental circuits in the final groups differs from the above. The total final group load is assumed to be a combination of the estimated over flow traffic (a residue from the HU circuit analysis) plus the first-route traffic estimated for the final group. The inverse Poisson blocking formula[17] is used to estimate an overall engineering load:

$$L'(IX) = \alpha\bigl[l - P(n'-1, \alpha)\bigr] + n' \cdot P(n', \alpha),$$

where

$$P(n', \alpha) = \sum_{x=n'}^{\infty} \frac{\alpha^x e^{-\alpha}}{x!}.$$

These are familiar telephone traffic formulas. As with the HU groups, the state engineering loads for the final groups are first updated and then revised to reflect changes in the load variables which result from other effects such as price revisions.

The procedure, involving combining the overflow traffic with the direct offered load to determine estimates of the final group engineering loads, is to compute:

[17] This formula is based on an assumption that blocked calls which fail to find an idle path through a facility will wait for a period equal to their normal holding time and then disappear from the system.

(a) $\sum$ overflow traffic from all HU groups for network (N) at the state network busy hour

$$= I(IX)HU\ OU_{.\hat{\jmath}.N}$$

(b) $\sum$ overflow traffic from all HU groups for network (N) at the new network busy hour, based on revisions

$$= I(IX)HU'OU_{.\hat{\jmath}'.N}$$

Then, using the direct offered load for a final group for network (N) at the state network busy hour at $L(IX)_{.\hat{\jmath}.l,}$ and the direct offered load for a final group for network (N) at the new network busy hour, $L(IX)'_{.\hat{\jmath}.l,}$, the following change can be computed:

$$PI(IX)F_{.\hat{\jmath}'.N} = \frac{L(IX)'_{.\hat{\jmath}'.l} + I(IX)HU'\ \ OU_{.\hat{\jmath}'.N}}{L(IX)_{.\hat{\jmath}.l} + I(IX)HU\ \ OU_{.\hat{\jmath}'.N}}.$$

Estimates of revised engineering load are then computed as

$$L(\epsilon)(IX)_{.\hat{\jmath}'.l(F)} = L(\epsilon)(IX)_{.\hat{\jmath}'.l}(PI(IX)F_{.\hat{\jmath}'.N}),$$

based on the $\hat{\jmath}'$ network busy hour. These load estimates are used to estimate the circuits ($n' \ldots_{lF}, n'' \ldots_{lF}$) required to meet a specified grade of service using the Poisson blocking formula.

The incremental circuit requirements (Δn) computed and summarized by various categories specified by the analyst,

$$\Delta n \ldots l_{(F)} = \left(n'' \ldots l_{(F)} - n' \ldots l_{(F)}\right).$$

This entire procedure is carried out on a circuit group basis, and recognizes the change in network characteristics which occur as a result of changes in demand.

The above analysis makes the basic assumption that the additional load will not create a need for establishing a significant number of new high usage groups. The cost effect of this approximation was investigated by analyzing the incremental Bell System costs associated with a theoretical network allowing new high usage groups to be established for economic CCS values varying between 14 and 26. Within this region, it was determined that only small errors derive from this approximation.

Incremental transmission facilities, Part 2 of the analysis. The next part of the intertoll circuit requirements model estimates the incremental transmission circuit facilities which will minimize total network costs over the planning period, subject to multicommodity flow requirements and concave link cost functions. Procedures to derive an exact solution are extremely complex, both because of the size of the problem and because of the concavity of the cost functions. Therefore, an approximate solution is derived through use of the following procedure.

As defined earlier, the transmission facilities associated with circuits between two switching points may consist of carrier systems, coaxial cables, radio (microwave) facilities,

or some combination of these facilities. When designing the circuit plant and choosing from this array of transmission facilities, the engineer selects the most economical con figuration in terms of serving both immediate and future needs.

Circuit requirements, once determined in Part 1 of this model, must be related to *sections* of facilities which comprise the intertoll routes at present or which will be installed to serve future demand. The base (*i.e.*, pre-incremental) circuit facilities between switching points are made up of various sections of transmission plant which exist on different physical routes, having been constructed at different points in time in order to meet varying needs and reflecting technology current at the time of their installation. Thus, the circuits connecting two non adjacent switching points consist of *facility sections*, and the incremental facility program must be related to these sections in order to make it possible to estimate transmission facilities that will be representative of what would be added to serve the incremental traffic and still be consistent with facilities that would serve all future demand for the planning period.

The following are the variables used in this portion of the model:

(1) *Exogenous variables*

(a) Preliminary cost functions by major facility class.

These are the cost functions for facilities (including future technology) during the planning period.

(b) Discount rate or cost of capital.

This is the rate used in capital budgeting to evaluate alternative investment programs in order to select the facilities which represent the minimum cost strategy.

(c) Circuit requirements.

These are the incremental network circuits as estimated in Part 1 of the analysis.

(2) *Status variables and system parameters*

(a) Mapping function.

This function maps the incremental circuit estimates onto a planning model structured to give facility transmission sections that will permit incremental transmission facilities to be estimated.

(b) Base state information on the model network of circuit facility sections.

These data include types of facility, total used capacity, and available capacity at a base point in time (t_0).

(c) Growth strategies.

These are the growth strategies for the planning period based on the present rate plans and are developed external to the analysis for each facility section in the basic planning model. This growth is a function of all System services, since the planning

of facilities is done for total demand (*i.e.*, switched services, private line services, audio and video services, etc.). Total demand must be analyzed in evaluating the routing and selection of types of transmission facilities to give efficient (minimum long-run) costs over the planning period.

(d) Technology data.

This information defines and specifies the technology that will be available for facilities during the planning period.

(3) *Endogenous variables*

(a) Quantities of transmission facilities by type. These data comprise the number of intertoll circuits to be added on intertoll facilities, by type of facility and by timing of installation, in order to serve changes in demand over a planning period at a minimum investment cost.

(b) Preliminary investment costs. These represent the preliminary capital costs associated with the facilities in (a), expressed as present worth of yearly investment.

The basic flow of information through the second part of the intertoll circuits model is shown in Figure 35. A first step in estimating the incremental transmission facilities associated with the estimated incremental circuits is to map the circuit requirements generated by Part 1 onto an aggregate transmission planning network consisting of a set of geographical planning nodes. Through this mapping procedure, circuits are mapped into internodal facilities and into intranodal facilities. Internodal facilities are for the most part the long-haul circuits and are made up of facilities which have large capacity, such as the L3 and L4 coaxial systems, the TD-2, TD-3, and TH radio systems, and so on. Intranodal facilities are primarily made up of short-haul circuits — N carrier, T1 carrier, TJ, TM, and the like — and their use is usually associated with routes less than 200 miles long.

The basic components of this planning network model of the long haul transmission network were developed by the Fundamental Planning Group of the Long Lines Department at AT&T. Figure 36 is a map showing the 88 nodes and 148 links which comprise the aggregation. Each of the 88 nodes represents many terminal locations within an area, and the circuits connecting those locations comprise the intranodal sections; for example, the New York node represents not only the metropolitan New York area but also Newark, White Plains, Morristown, and so on. The transmission facilities connecting the 88 nodes comprise the internodal sections. All transmission facility sections, or links, represent actual circuits in existence between node pairs.

The incremental circuit requirements are mapped into this aggregate planning network in two steps. First, each toll center area (TCA) is mapped to a unique node in the planning network (assignment being based upon minimum distance between TCA and node). Circuit groups which have originating the terminating TCAs associated with them are then mapped onto the planning network according to the TCA node mapping. Thus internodal and intranodal circuit requirements are developed.

Second, a routing table has been developed which is used to route (that is, assign) each

internodal circuit group over a unique path interconnecting the 88 nodes; the path selected represents the minimum-cost path, based on minimum link mileage. This method provides a good approximation of minimum-cost routing, because minimum mileage is related to minimum first cost expressed in dollars per mile.

FIGURE 35

FACILITIES REQUIREMENTS AND INVESTMENT ANALYSIS, INTERTOLL CIRCUIT FACILITIES MODEL

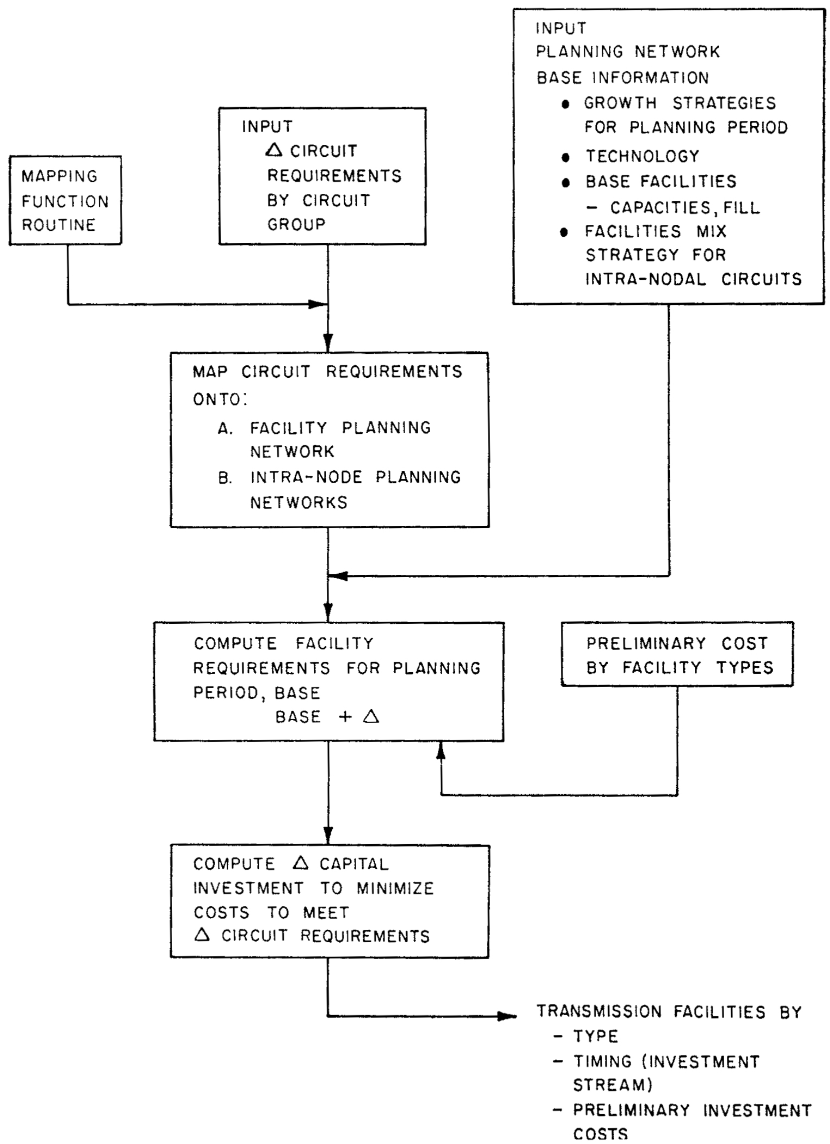
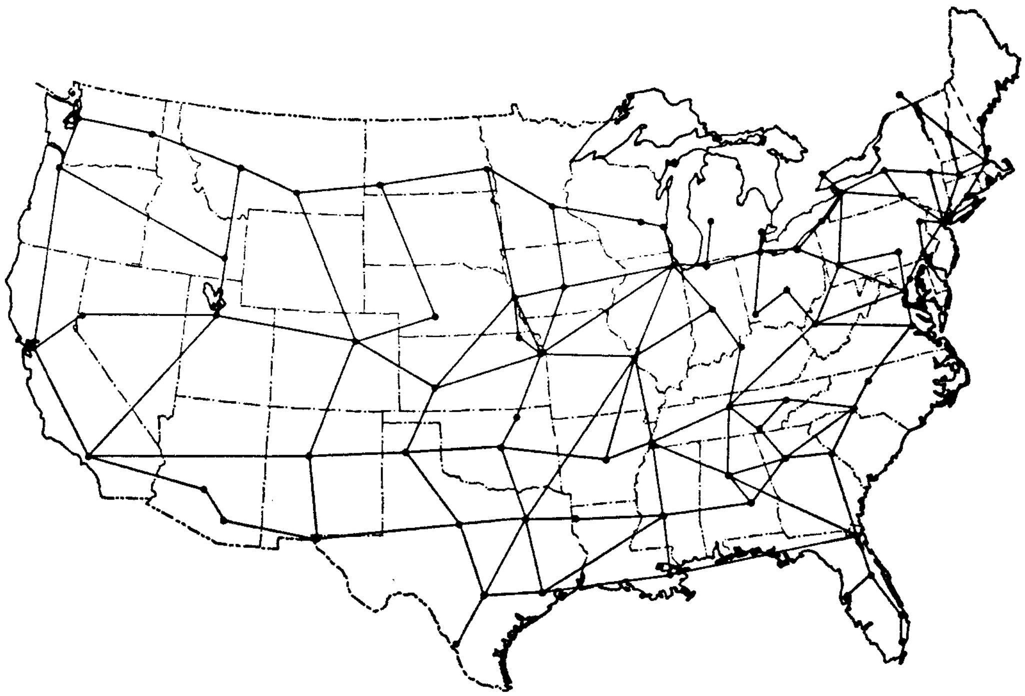


FIGURE 36

PLANNING NETWORK MODEL OF LONG-HAUL TRANSMISSION FACILITIES



Internodal facilities. Now the internodal incremental circuit requirements between TCAs are mapped over the internodal minimum-mileage links. When this has been done, the circuit demand Δn_{ij} which exists between toll centers i and j has been associated with planning nodes A (which includes TCA i) and B (which includes TCA j), and Δn_{ij} has been assigned to network link AB.

For each internodal link, the following information is available by *facility sections:*

(a) Type of transmission systems,

(b) Total capacity,

(c) Authorized capacity at time t_0 (*i.e.*, authorized channels),

(d) Equipped capacity at time t_0,

(e) Total working circuits, both in use and spare, and

(f) Growth factors for the planning period.

Together, these data characterize the state of the network at time t_0. In order to analyze the incremental effects on future network planning, it is necessary to quantify the future status of the network — *i.e.*, to study demand over a planning period, determine the date on

which excess capacity will have been used up, and select an optimum strategy for making additional transmission facilities available on each link in order to serve the incremental circuit demand plus the expected growth from the exhaust date to the end of the planning period, while still minimizing cost. This procedure is performed by a sub-model which treats each link, considering the time from exhaustion of spare capacity, T_e, to the end of the planning period, T_p. Estimates are based on minimum investment associated with each link, and the transmission systems considered include both existing systems and those which may become available during the planning period.

This initial model has the following limitations worth noting:

(a) The model chooses an optimum strategy for each link as an independent entity. Rearrangement of demand is not presently considered.

(b) The optimization is based on minimum link investment, due to the complexity of the cost function, and does not include terminal equipment costs.

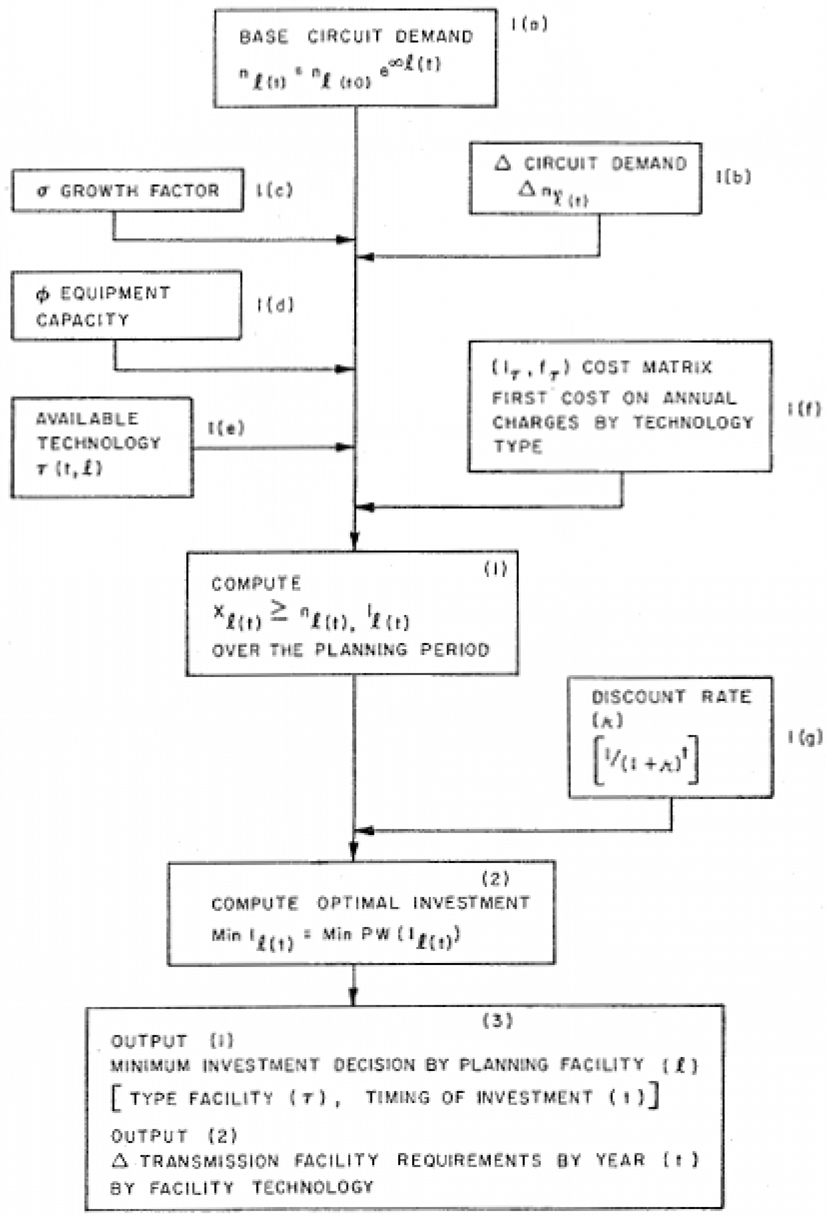
(c) The transmission systems considered include existing systems, as well as those which will become available during the study period. Therefore, system cost functions vary in reliability.

Figure 37 is a diagram showing the operational characteristics of the internodal portion of the analysis. Given the inputs $I(i)$ which consist of: $I(a)$ the initial circuit demand capacity $\eta_{l(t)}$ between planning nodes; $I(b)$ the incremental circuit demand $\Delta\eta l(t)$; $I(c)$ the projected growth rate (γ) of demand for each link; $I(d)$ the capacity factor for each link $\phi; I(e)$ the available technologies $\tau(t, l)$; and $I(f)$ a cost matrix (I_τ, f_τ) which gives first costs or annual charges by technology types (τ), the procedure (block 1) first computes for each link the first costs for different technologies that could be used to satisfy the requirements over the planning period. The algorithm then chooses (block 2) the transmission system from the available technologies $(\tau_{t,l})$ which minimizes the cumulative present worth of investments $PWI_{l(t)}$ at a pre-specified discount rate $(r)\big[I(g)\big]$.

After these investment decisions have been made for each link (l) in the network for each of the years in the planning period, the first cost and present worth of cost (investment and annual charges) are calculated for each year and accumulated over the entire study period. The outputs (block 3) are: (1) the first approximations to the cumulative present worth of investments and related facilities by transmission facility class, and (2) the Δ transmission facility requirements by technology class by timing of investment (year t).

FIGURE 37

AMPLIFIED BLOCK DIAGRAM, INTERTOLL CAPITAL INVESTMENT MODEL



NOTE: I(x) = INPUT DATA

Intranodal facilities. The intranodal model, or that portion of the intertoll facilities model which deals with intranodal circuits, has been structured along two paths. First, a procedure similar to the internodal model was researched. However, preliminary studies

indicated that the development of status information for the major intranodal transmission facilities (by sections) within each of the 88 TCA areas was a large undertaking and beyond the present scope of this analysis. It is still hoped that a smaller subset of areas can be developed by clustering of nodes, or that some other aggregation can be used that will serve as a representative set of nodes for this portion of the analysis. For the present, a more aggregate and simplified procedure is used to estimate facilities associated with the intranodal incremental circuits. The following assumptions are made:

(a) Incremental circuit requirements will not affect the *percentage mix* or *type* of short-haul transmission facilities included in present planning periods within mileage categories, but,

(b) The *quantity* of circuit requirements and the *timing* of their installation may change.

Based on these assumptions, the procedures consist of applying an aggregate System transmission facility mix strategy table (as an exogenous variable) to the intranodal incremental circuit requirements by length of haul. The strategy table provides estimates of the relative mix of transmission facility systems as a function of distance. This determination of prospective mix was developed from a differential analysis of interexchange circuit miles added during a study period and adjusted for future technologies. Field inventories provided in formation as to the types of facilities added by route miles, and were used in developing estimates of the probable mix of systems to be added during the planning period.

Incremental mixes for combined L- and R-type facilities (which are short-haul types of circuits) were developed based on an analysis of the Bell System's construction program. Carrier circuits (N- and T-type) were separated from line haul facilities, so that the latter would be available for further analysis; for carrier circuits, a differential facilities mix was used as a proxy and was related to the L- and R-type mix in order to develop percentages of each type of facility that would be installed to satisfy requirements.

The endogenous variables produced by the intranodal model comprise the quantities of incremental circuits and incremental circuit miles, by type of transmission system.

The incremental mix strategies are now applied to the incremental circuits for each of the 88 intranodal areas. This gives the incremental route miles and incremental numbers of circuits by node identifier (k) and by type of transmission facility (τ).

The model is:

$$\Delta n_{k\alpha\tau} = \Delta n_{k\alpha}\omega_{1,\tau},$$
$$\Delta m_{k\alpha\tau} = \Delta m_{k\alpha}\omega_{2,\tau},$$

where

$\Delta n_{k\alpha\tau}$ = incremental circuits for node k, route mileage band α, circuit type τ,
$\Delta m_{k\alpha\tau}$ = incremental circuit miles for node k, route mileage band α, circuit type τ,
$\omega_{1,\tau}$ = System facility mix distribution for circuits of τ type, and
$\omega_{2,\tau}$ = System facility mix distribution for circuit mileage of τ type circuits.

For the System,

$$\Delta n_{\cdot\cdot\tau} = \sum_{k,\alpha} \Delta n_{k\alpha\tau},$$

$$\Delta m_{\cdot\cdot\tau} = \sum_{k,\alpha} \Delta m_{k\alpha\tau},$$

where

$\Delta n_{\cdot\cdot\tau}$ = total incremental circuits of transmission type τ, and

$\Delta m_{\cdot\cdot\tau}$ = total incremental circuit miles of transmission type τ circuits.

Intertoll capital investment, Part 3 of the analysis. In the third and last part of the intertoll circuit facilities analysis, cost functions are applied to the incremental transmission facilities estimates for the planning period. This procedure refines the estimates which were used in Part 2 to select minimum-cost transmission facilities.

The study of price-cost relationships involves comparing the costs of various alternative investment strategies. Since one future cost is the repayment of new capital commitments, the time value of money must be considered in the investigation of alternatives. To represent the average long-run incremental costs associated with the estimates of transmission facilities requirements derived in Part 2 for the planning period, present worth at the base (test) year of the investment is computed. This is done by first applying cost functions to the facilities requirements, then applying compound interest factors to the expenditures to obtain their present worth at a common point in time (equivalent expenditures) or at the same period of time (equivalent annual expenditures).

The following are the variables used in this model:

(1) *Input variables*

  (a) Estimated quantities of incremental transmission facilities.

  These facilities requirements are input (hence module-exogenous), and are supplied by facility type and by time of installation.

  (b) Cost functions.

  These data are given by type of transmission facility.

  (c) Cost of capital data.

  These are discount factors used in present worth calculations.

(2) *Endogenous (output) variables*

(a) Estimated quantities of transmission facilities and associated long-run incremental costs.

This output is summarized by five account categories (η), by facility type (τ), by circuit type (β), by route mileage band (α), and by timing of the investment (t). All subscripts are defined in Table 10. The account arrays computed represent the present worth of capital investment in facilities in each of the subscripted categories.

TABLE 10

DEFINITION OF COST ARRAY SUBSCRIPTS

LET h = INDEX ON INTRANODAL AREAS,
ℓ = INDEX ON LINKS, (INTERNODAL)
α = ROUTE MILEAGE BAND NOTATION (1....10)
β = CIRCUIT TYPES (1–4):
1. INTEREXCHANGE (IX) (INTERNODAL)
2. INTEREXCHANGE (IX) (INTRANODAL)

τ = TRANSMISSION FACILITY TYPES
δ = 1, MILES OF CIRCUITS
δ = 2, NUMBER OF CIRCUITS
η = 1, OSP – OUTSIDE PLANT (LINE HAUL) (ACCTS. 241–244)*
η = 2, 57C – CENTRAL OFFICE CIRCUIT EQUIPMENT (ACCTS. 221–57C)*
η = 3, 67C – CENTRAL OFFICE RADIO EQUIPMENT (ACCTS. 221–67C)*
η = 4, LAND ASSOCIATED WITH CIRCUIT EQUIPMENT (ACCT. 211)*
η = 5, BUILDINGS ASSOCIATED WITH CIRCUIT EQUIPMENT (ACCT. 212)*
t = TIMING OF INVESTMENTS – t_1, t_2, ... t (PLANNING PERIOD.)+

*DESIGNATIONS USED TO RELATE INVESTMENT TO THE FCC SYSTEM OF ACCOUNTS.

+ NOT UTILIZED IN INTRANODAL INVESTMENT PROCEDURES

Internodal circuit investment cost functions. The costs for L- and R-type circuits were derived from a capacity analysis of additions as estimated in the Bell System construction program, and represent incremental investment. The N- and T-type circuits costs were based on recent construction data obtained from Bell System Operating Companies. All costs were categorized by facility type: (1) high-capacity microwave radio relay and coaxial systems, (2) low-capacity line portions of cable carrier systems and low-capacity microwave radio relay systems, and (3) other elements of line haul equipment (master-group and J-group terminal equipment, intermediate carrier terminal equipment, and various types of connectors).

TABLE 11

SYSTEM CAPACITIES

TYPE SYSTEM	FREQ. BAND	RADIO CHANNELS (2-WAY) SERVICE	PROTECTION	MASTERGROUPS (2-WAY) SERVICE	PROTECTION
TD2	4GHz	10	2	10	2
TD2B	4GHz	10	2	20	4
TD3	4GHz	10	2	20	4
THI	6GHz	6	2	18	6
TH3	6GHz	6	2	18	6

Table 11 shows the system capacities for high-capacity radio relay and coaxial systems. Costs were developed for each two-way master group mile; a mastergroup consists of 600 voice-frequency channels multiplexed together and sharing certain equipment on broadband carrier systems. Table 12 shows system capacities for coaxial broadband transmission facilities, for which costs were also developed on a two-way mastergroup basis. All of these costs were developed in two categories — "getting started" costs and variable costs.

TABLE 12

SYSTEM CAPACITIES

TYPE SYSTEM	NUMBER OF COAXIALS SERVICE	PROTECTION	MASTERGROUPS (2-WAY) SERVICE	PROTECTION
L4	10	2	30	6
L4	18	2	54	6
L5	10	2	75	15
L5	18	2	135	15
L5	20	2	150	15

"Getting started" costs are the costs of plant (land, buildings, towers, antennas, and so on). Cost functions for land and buildings were derived from data developed by AT&T's Engineering Economics Department based on Operating Company-reported information, and pertain to all land and buildings which, during the study period, housed switching equipment or circuit equipment, or had been re located to make room for such equipment, or were destined to house such equipment in the future. The costs do not apply to properties such as business office buildings in the Bell System, which contain no switching equipment. They may, however, apply to business office buildings which are destined to hold switching equipment at some planned future time. From these data, incremental investment factors based on new buildings and building additions constructed over the three years preceding the study period were developed.

The second cost category, variable costs, pertains to equipment required to establish specific radio channels, or to establish a pair of coaxials, etc., such as line repeaters, line

regulators, and the like. For some radio systems (TD2 and TH1) these costs were based on field record data, while for other radio systems and all coaxial transmission systems the costs were developed by applying engineering estimates to existing data.

Costs for newly designed facilities were based on the latest engineering knowledge, and where elements of existing systems were found to be similar to elements of systems under development, the costs of these elements were analyzed and modified to reflect future price trends and changes in design and then used for the new facilities.

The development of costs for low-capacity systems was similar. Costs were developed separately per mile of cable conductor for aerial, underground, and buried construction and then developed into composite costs on the basis of relative expenditures for each such type of construction. These costs were expressed as a function of revenue producing circuit mile by the application of fill factors, or factors which show the percentage of cable capacity used to serve demand. Other low-capacity system costs were developed through referencing construction program records.

For elements of high-frequency line facilities, such as mastergroup multiplex equipment, J-group multiplex equipment, and the like, the amounts includable in figuring line haul mileage costs depend on the estimated unit costs of the items and on the number of such items required. The costs and their respective occurrence rates were developed from an analysis of the Long Lines broadband facility program.

Intranodal circuit investment cost functions. The simplified analysis used by the intranodal facilities pending development of a more complete data base required a different costing procedure, because the analysis had been based on the assumption that — for the class of plant to which the analysis applies — the incremental change would not affect the percentage mix (type of plant facilities planned in the construction program, by route mileage band). This is equivalent to assuming a constant growth rate, once the level of change is determined. However, the timing can be affected for the class of plant.

FIGURE 38

ASSUMED LINEAR GROWTH PATTERN FOR INTRANODAL CIRCUIT COSTS PROCEDURE

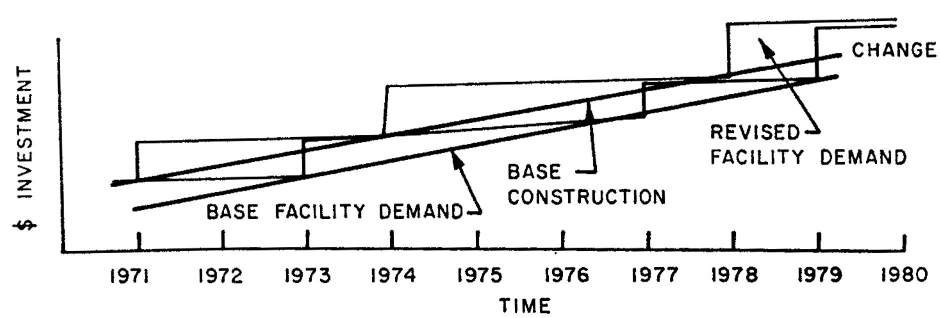
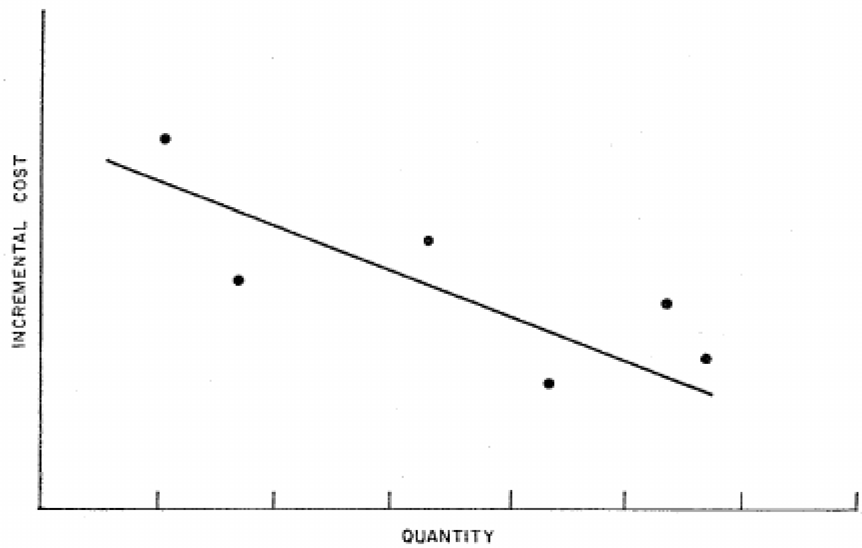


Figure 38 is a diagram of the above assumption, based on a simple linear growth pattern. The costing procedure based on this assumption reflects the time value of money.

FIGURE 39

COST VS. QUANTITY



Data were obtained from Long Lines to give the dollar amounts of expenditures planned for construction in each of the years through 1980, by facility class components. In each year, the total expenditure for each major class of facility was calculated. Total costs were plotted for the years through 1980, and a regression analysis was applied by fitting, by least-squares, a power curve of the general type $y = ax^b$. After the cost curve was developed, differential line facility expenditures for each year were calculated. For the computed differentials for each year in the study planning period, discount factors were applied to express the total dollar amount of additions for the period in terms of their test year value (for example, 1971). This permitted an average incremental cost to be developed by facility class, to reflect the timing of the investment for the planning period discounted back to a base year. This analysis was repeated for various levels of construction, where growth was held constant and coefficients were developed to reflect the effects associated with the timing of the different investment streams.

Graphs of cost versus quantity were then derived. Figure 39 is a typical example of these results. The values developed, reflecting the relationships of cost to quantity, were applied to the capital investment in each of the given categories of plant.

Application of cost functions. In the final phase of the intertoll circuits model, the appropriate cost functions are applied in a straight forward manner; the results are the incremental interexchange facilities investments associated with a change in demand. These investments are calculated on the basis of present worth during a given test year, and represent average long-run incremental investment. The following two calculations summarize the procedures:

(1) *Internodal investment costs:*

$$INV(IX)^{\eta}_{l\tau t} = \left[X^{\delta}_{l\tau t}\right] * \left[c^{\delta,\eta}_{\cdot\tau\cdot}\right],$$

where

$INV(IX)^{\eta}_{l\tau t} =$ capital investment costs for planning link index l, facility type τ, timing investment t, account code η (1 = outside plant, 2 = central office circuit equipment, 3 = central office radio equipment, 4 = land associated with intertoll equipment, and 5 = buildings associated with intertoll equipment),

$X^{\delta}_{l\tau t} =$ quantity of facilities in terms of equivalent number of circuits ($\delta = 1$), or quantity of facilities in terms of route miles ($\delta = 2$), and

$C^{\delta,\eta}_{\cdot\tau\cdot} =$ cost function for facility class τ, account code η, for $\delta = 1$ and 2 as above.

The output of this multiplication is a vector of costs for each link, by time of investment. These values are then discounted back to the base period by computation of present worth:

$$PWINV(IX)_{l\tau\cdot} = INV(IX)^{\eta}_{l\tau t}(1+\tau)^{-t},$$

and summed over transmission facility types and links to give estimates of long-run incremental costs by account codes:

$$LR[INV(IX)^{\eta}\cdots] = \sum_{\tau}\sum_{l} PWINV(IX)^{\eta}_{l\tau\cdot}.$$

(2) *Intranodal investment costs:*

$$INV(IX)^{\eta}_{k\tau\cdot} = \sum_{\alpha}\left[X^{\delta}_{h\tau\cdot\alpha}\right]\left[C^{\delta\eta}_{\tau\cdot\alpha}\right]\left[\beta_{\tau}\right],$$

where

$k =$ intranodal circuit group code,

$\alpha =$ route mileage band (different cost functions having been developed by route mileage class to reflect additional required repeaters, etc),

$\tau =$ facility class,

$C^{\delta\eta}_{\tau\cdot\alpha} =$ average incremental cost by facility class τ, account code η, for circuits ($\delta = 1$) or route miles ($\delta = 2$), discounted to the base year, and

$\beta_{\tau} =$ coefficient to adjust the average incremental cost $C^{\delta\eta}_{\tau\cdot\alpha}$ for the level of equipment installed.

These values are then summed over nodes and transmission facility classes to estimate long-run incremental investment by account code for the short-haul (intranodal) circuits.

Module 5 : Toll connect facilities cost analysis. The costs of the toll connect circuits that will be needed to handle increments in telephone demand are estimated by this supply sub-model. This particular analysis deals with only that portion of the additional traffic which is assumed to be carried on circuits connecting the Class 5 end offices to Class 4 and above toll centers. (See Section 5, in which the toll center network hierarchy is described, particularly Figure 7.) These circuits provide access from each subscriber's end office (local exchange) to the toll network. Within this category, circuit groups carrying various types of calls (operator-handled, direct dialed, or coin) are engineered separately, and, similarly, those carrying one-way originating, one-way terminating, or two-way traffic are processed separately. The reason for this separation is that there are special requirements for these types of traffic which are dealt with separately in order to maximize equipment utilization and thereby minimize costs.

Network structure for toll connect circuit groups. For this sub-model, a network is defined as all the links running into or out of a toll center (Class 4 or higher) to or from end offices (Class 5). As discussed in Section 5, a toll connect circuit group may be either high usage (*HU*) or final, which determines the engineering schedule of the particular route. However, the majority of the toll connect trunk groups are not engineered as high usage, but rather on a final basis.

Data. The data sets relevant to this portion of the model are the following:

(1) *Status variables*

Two sources of status data are used for this analysis:

A. A toll connect circuit inventory, containing the following in formation on each toll connect circuit group —

1. End points of the group (toll center and end office),
2. Engineered level of circuits in the group,
3. Engineering schedule of the group (HU or final),
4; Direction of the group (one-way in, one-way out, two-way),
5. Type of traffic carried (operator-handled, DDD, coin), and
6. Route miles of the group.

B. A set of traffic load "profiles" — information given in hourly periods of the day, which shows the distribution of the offered load for the toll center network. (Information was not available on individual circuit groups, as was obtained for the intertoll circuit group analysis.) This set of profiles is used as the "status" load distribution — *i.e.*, the distribution of base traffic.

(2) *Exogenous (input) variables*

The following input variables are used relative to the toll connect trunks:

A. Grade of service variables,

B. Engineering schedules (as discussed above), and

C. Cost ratio curves, used to compute the economic CCS associated with HU groups.

An additional category of variable was used which can be considered exogenous with respect to the toll connect model, but endogenous to the overall demand-supply analysis. These data are the incremental hourly offered loads to the toll connect circuits by major service, which were generated by the demand forecast models and translated in the translation phase.

(3) *Endogenous (output) variables*

The following output variables are generated —

A. Incremental toll connect circuit requirements, which can be summarized by type (*i.e.*, DDD, operator-handled, coin), by length (distance), by density (size of circuit group), and by geography,

B. Incremental toll connect circuit mile requirements (which can be summarized as above),

C. Incremental toll connect circuit transmission facilities requirements, summarized by type and geography if desired, and

D. Incremental toll connect circuit capital costs, summarized by capital investment accounts conforming to accounting classifications set up by the FCC.

Model logic. The procedure for estimating capital costs associated with incremental toll connect circuit requirements is partitioned into three sections similar to the method employed within the Intertoll Circuit Analysis discussed earlier. The first part is an analysis in which the incremental toll circuit requirements are evaluated for a particular demand change. The second part calculates the transmission facilities which would be required over the planning period, and the third computes an estimate of the capital investment which would result.

Since load profiles could not be derived easily on a per-group basis for the toll connect circuit groups, the assumption was made that the toll center area (TCA) base profiles and the incremental profiles by type and class (DDD, operator, coin) of service could be used as average estimates of toll connecting circuits to evaluate circuit group loads before and after a demand change. These estimates are used as a basis for engineering the toll connect circuits. Since the majority of toll connect circuit groups are engineered as final rather than as high usage trunks, alternate routing (including the non-coincident and overflow traffic) was not considered to be a significant problem, and average network profiles should give reasonable estimates.

The first step in the analysis is to compute revised TCA profiles (based on either updated state variables or the price change) for originating, terminating, and through traffic (T), taking into account traffic characteristics (major service classification, rate mileage step,

demand characteristics, and hourly time of day periods). Thus

$$I(T)_{ijs\eta} = L(T)_{ijs\eta}\frac{B'_{ijs}}{B_{ijs}}$$

where

$I(T)_{ijs\eta}$ = the increment for TCA$_\eta$, rate mileage band i, period j, service s, and type (T) — originating, terminating, two way, or combined traffic groups,

$L(T)_{ijs\eta}$ = base load in CCS for the same categories,

B'_{ijs} = projected System CCS load for toll connect trunks in period j, rate mileage band i (from the translation module), and

B_{ijs} = base System CCS load for toll connect trunks for the same categories.

Then the fractional change $\left[PI(T)_{.js\eta}\right]$ in CCS load during period j, for service s, TCA_η is

$$[I(T)_{.js\eta} + L(T)_{.is\eta}]/L(T)_{.js\eta}.$$

The hourly revised profile for TCA_η in period j is then computed as

$$L(T)'_{.j.\eta} = \sum_s \left[(L(T)_{.js\eta}PI(T)_{.js\eta}\right].$$

A change in the busy hour load for each TCA is now computed as the ratio of the new busy hour load to the old busy hour load:

$$PL(B)_{...\eta} = \frac{L(T)'_{.j'.\eta}}{L(T)_{.j^*.\eta}}$$

where

$$j^* = \text{old busy hour, and}$$
$$j' = \text{new busy hour.}$$

The offered load for combined circuit groups is estimated by summing the originating and terminating traffic for a toll center area.

To compute incremental trunks, the engineered number of circuits (n) for each group is determined from the toll circuit group inventory file. The offered engineered load (a) which would require (n) circuits (taking into account the Bell System service criterion of blocking) is then computed in a procedure similar to that discussed in the intertoll circuit section, using the engineering procedures and formulas in reverse. The result is assumed to be the designed engineered load for that group.

The new peak load for the engineered period is now estimated using the ratio of the peak hour loads for the base and revised traffic (including the increment) as computed above. The revised engineering load (a') is then computed by:

$$a' = a * PL(B)_{...n}(T).$$

The number of circuits necessary to handle a traffic load of a' is then determined, as previously described for the intertoll model, and the incremental circuit estimate is computed using a procedure similar to that used in the intertoll model:

$$\Delta n = n' - n,$$

where

$$n = \text{number of circuits required for load } a,$$
$$n' = \text{number of circuits required for load } a'.$$

This procedure is used to update the state variables and to analyze load changes due to price changes.

The final groups are assumed to be engineered to provide .01 blocking probability, and the high usage groups are assumed to be designed on an economic basis (ECCS) where, as stated previously, trunks are added to the direct route until the cost per CCS carried by the last trunk added is equal to the cost per CCS carried by the last trunk added on the alternate route.

The output of this phase of the toll connect sub-model is the incremental toll connect circuit requirement, and the incremental circuit miles associated with the incremental demand.

Facilities requirements. In the second phase of the toll connect analysis, the incremental transmission facilities that would be required to serve the demand offered to toll connect circuit groups are estimated. The procedure used at present is similar to that used in the intertoll trunking *intranodal* sub-model.

Basically, a System aggregate facility mix strategy table is applied to the incremental toll connect circuit requirements as a function of length of haul and density. The underlying assumption in this procedure is that the *timing* of facilities investments will be affected for these rather short-haul circuits, but not the basic *distribution* or *mix.* This approach for determining prospective circuit facilities mix was developed from a differential analysis conducted during a given study period and adjusted for future technologies.

Toll connect facilities costs. In the last phase, cost functions derived in a manner similar to that used for the intertoll model cost functions, and described in the previous section, are applied to the estimated new toll connect transmission facilities. This produces an incremental cost figure associated with serving the new or incremental segment of demand. The procedure for this calculation is once again the same as that used for estimating intertoll intranodal circuit investment costs, and is described in the foregoing section.

Briefly, it consists of applying average incremental toll connect transmission facility costs by class of facility, where coefficients are used to adjust for the timing of the investment over a planning period so that, in effect, the investment stream is discounted back to the test year. These coefficients are applied to the capital investments by category of plant, thereby reflecting the relationship of cost to quantity.

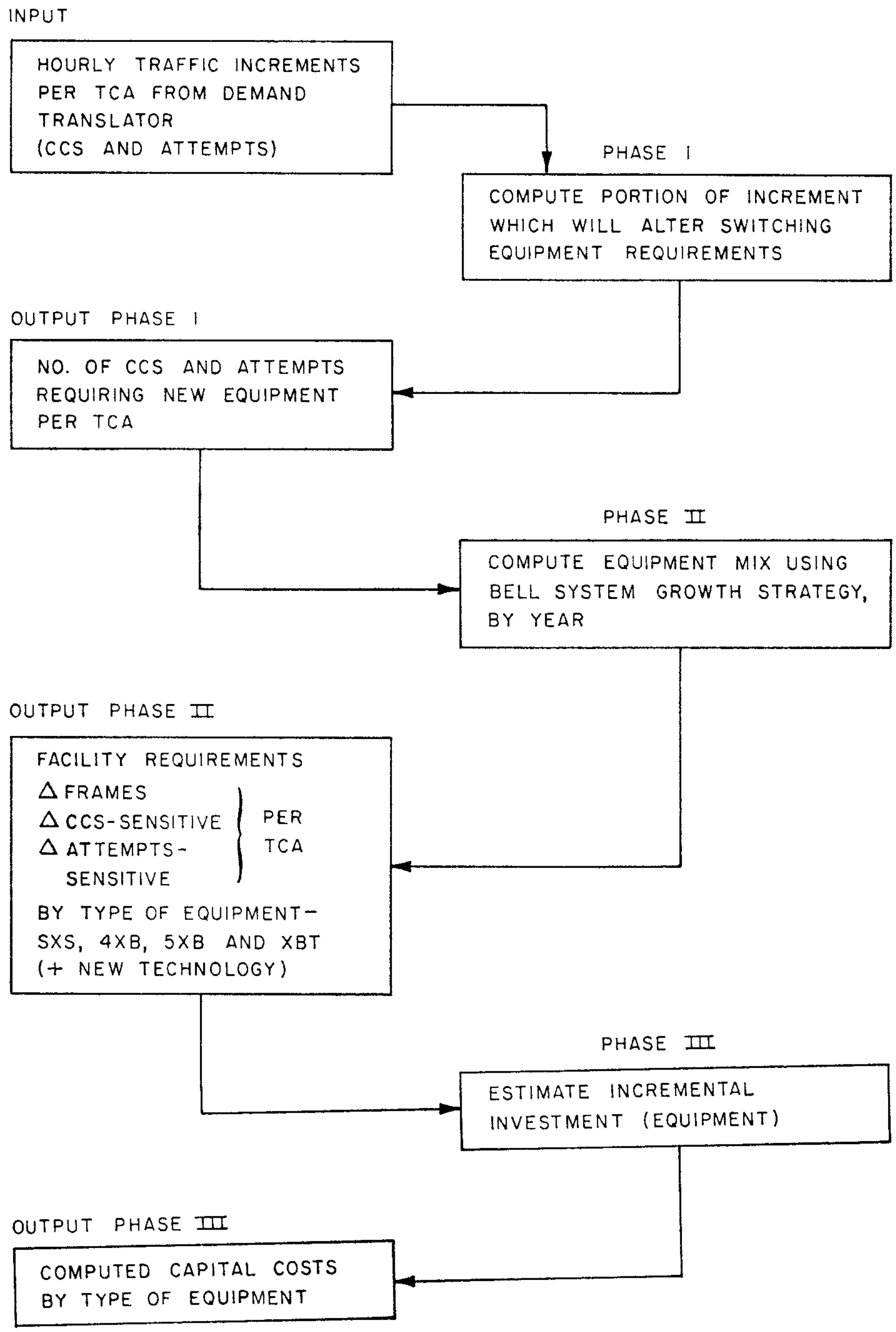
Procedures are being researched to develop a more refined toll connect transmission

facility planning model that will not require the assumption of an unchanged mix of transmission facilities. The number of areas and the variations among areas together with the need to gather status data on the transmission facilities have delayed completion of this effort. However, the present procedure is felt to give reasonable approximations of average long-run transmission facilities and costs.

Module 6: Toll dial switching facilities cost analysis. The toll dial switching module estimates the incremental switching equipment which is required to serve the structured incremental demand which is forecast by the demand models and adapted by the translation sector. Some discussion of this switching equipment was included in Section 5; the manner in which the model estimates both the incremental equipment required and the associated capital investment will now be described. The problem consists of determining first the quantities and types of equipment that would be needed to serve the additional CCS and attempts, and then costing out these new equipment acquisitions so that the effects on capital expenditures for switching can be estimated.

FIGURE 40

OVERALL SIMPLIFIED BLOCK DIAGRAM,
TOLL DIAL SWITCHING FACILITIES COST MODEL



The dial switching equipment module — a sub-model — was designed to be executed in three phases. The first phase estimates the additional capacity in peak-load CCS and attempts for which new equipment would be needed; the second estimates the mix of equipment types which would be installed in Bell System switching centers to minimize the investment stream and still serve this load; and the third estimates the capital investment costs associated with the incremental pieces of equipment. Figure 40 is an overall block diagram of this module.

Data. There are module-exogenous, model-exogenous, status, and endogenous data associated with this sub-model.

(1) *Exogenous (input) variables*

The following input variables are exogenous to the sub-model:

A. Incremental offered load profiles (CCS and attempts) for 18 hourly periods of the day, by TCA. These module-exogenous profiles are similar to those used in the circuit facilities analyses, in that they show load during the day; however, in this case instead of measuring traffic on circuit groups, the profile shows load on switching equipment within the network. The offered loads are given in terms of attempts and holding time (CCS), since switching equipment is engineered on the basis of these two types of traffic plus trunk terminations. The incremental load profiles for the System are generated by the demand translation module, module 3.

B. Expected growth in switching requirements for each switching area during the planning period. This exogenous information has been determined as part of the planning program analysis for the System. The average annual growth characteristics are based on present growth rates by TCA for switching machines, developed from planning information and including:

—CCS, interstate traffic

—CCS, intrastate traffic

—attempts, interstate traffic

—attempts, intrastate and local traffic

C. A strategy table, giving the estimated apportionment of incremental peak load switching traffic (*i.e.*, the probabilities of load distribution) among major types of switching technology that will be added over the planning period.

D. Cost functions, by types of switching technology.

(2) *Endogenous (output) variables.*

The following are the output variables of the three phases:

A. Estimated peak load CCS and attempts requiring additional switching capacity, by TCA.

B. Estimated incremental CCS and attempts requiring additional switching capacity

to serve peak loads, by type of switching equipment and timing of investment for the planning period.

C. Estimated incremental capital investment by type of equipment and major accounting classification, discounted to the base period to represent long-run average incremental costs.

(3) Status variables.

The following are the status variables used to represent switching equipment in use as of a base period, and the load on that equipment:

A. Hourly profiles of attempts and holding time (CCS), representing the total switching load distribution and including all traffic handled by the equipment. The basic data were derived from the annual Point-to-Point Study[18] as average distributions of load for 18 hourly periods of the day over the ten-day period. Additional information in the form of total office marker register readings[19] was then used to estimate total load at each switching location. Finally, "through" toll traffic (toll calls neither originating nor terminating at a given machine, being carried as tandem traffic to another location) was estimated to adjust the base profiles for tandem CCS loads.

B. Switching Machine Inventory[20] information on each machine in existence, by TCA (the number of machines, the telephone company that owns them, and the type of equipment).

C. Average ten-high-day factors for both CCS and attempts, for each major switching machine in the inventory. Representative levels of CCS and attempts observed during the ten busiest days of the year are used in engineering equipment at the various switching locations in the network. Factors were developed by comparing these ten-high-day loads with the loads during the ten days of the Point-to-Point Study, in order to relate the study period levels to the engineered period levels.

D. Total office engineered capacity (CCS and attempts) for the busy season of the same year in which the study period used for the base occurred (to permit calculation of spare capacity in each office).

E. Estimated ultimate CCS and attempts capacity to which each TCA could be expanded (based on long-range Bell System plans).

F. Double-switching factors, used to reflect realistically the handling of traffic within multi-machine TCAs. Double switching of traffic often occurs in multi-machine areas, where the non coincidence of hourly traffic flow among the various pieces of equipment makes it efficient to route calls through more than one machine in an

[18] Data collected during ten days in March each year on point-to-point toll usage (messages and message minutes) to form a base for various demand and separations studies.

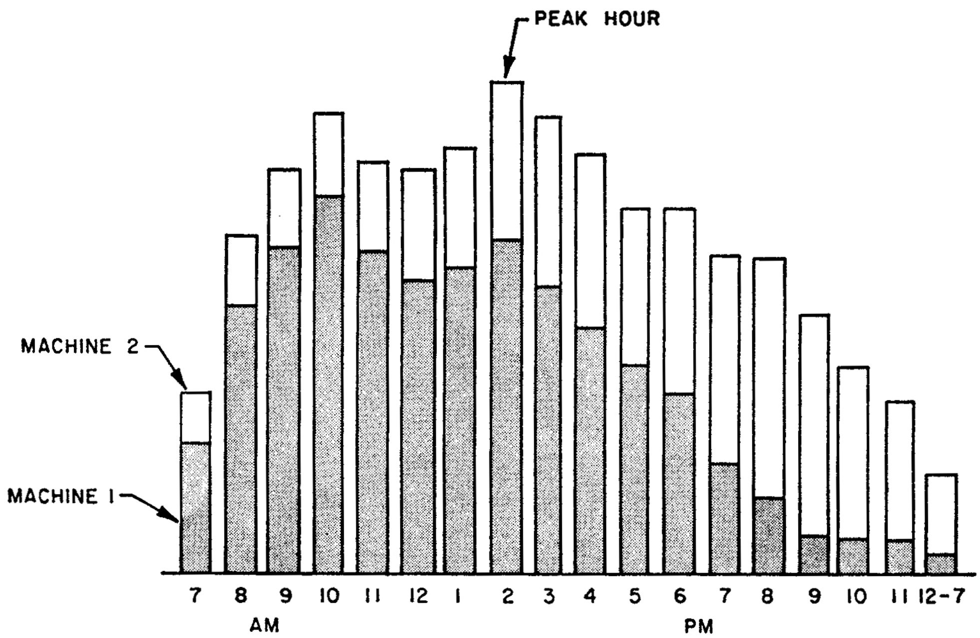
[19] A "peg count" of the total load on each switching machine is obtained by recording attempts on the part of the equipment called the marker.

[20] An inventory is conducted periodically of all equipment in use, and is made available for various studies by the Service Costs Department of AT&T.

area. Figure 41 illustrates this non-coincident traffic flow for one two-machine TCA. Note that although the busy hour for the TCA is 2 p.m., for the individual machines the busy hours are 10 a.m. (machine no. 1) and 8 p.m. (machine no. 2).

FIGURE 41

HOURLY PROFILE OF CCS LOAD FOR A MULTI-MACHINE TCA



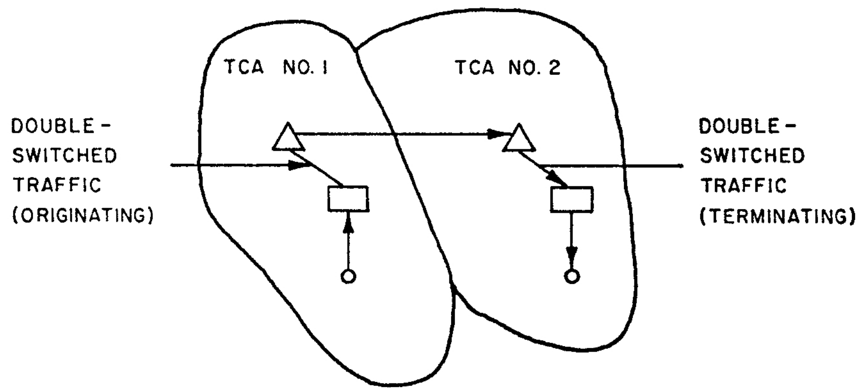
In serving the total TCA load, the equipment is most efficiently utilized through routing schemes which take advantage of this non-coincidence by overflowing traffic from a busy machine to a less busy or an idle one, thus causing the double switching. Figure 42 shows in a simplified form the double-switched load in two adjacent TCAs.

Double switching may occur for two other reasons: (1) the amount of traffic between a particular originating point and a particular terminating point may not warrant the provision of a direct trunk over the route, and (2) portions of the originating traffic which are direct-distance-dialed may have to be routed to toll machines equipped with AMA (automatic message accounting) equipment.

The double-switching factors were developed as percentages of the total originating and terminating loads on equipment in multi-machine TCAs, and were applied within the model to both the base and the incremental traffic.

FIGURE 42

DOUBLE - SWITCHED TRAFFIC LOAD, ADJACENT TCAS



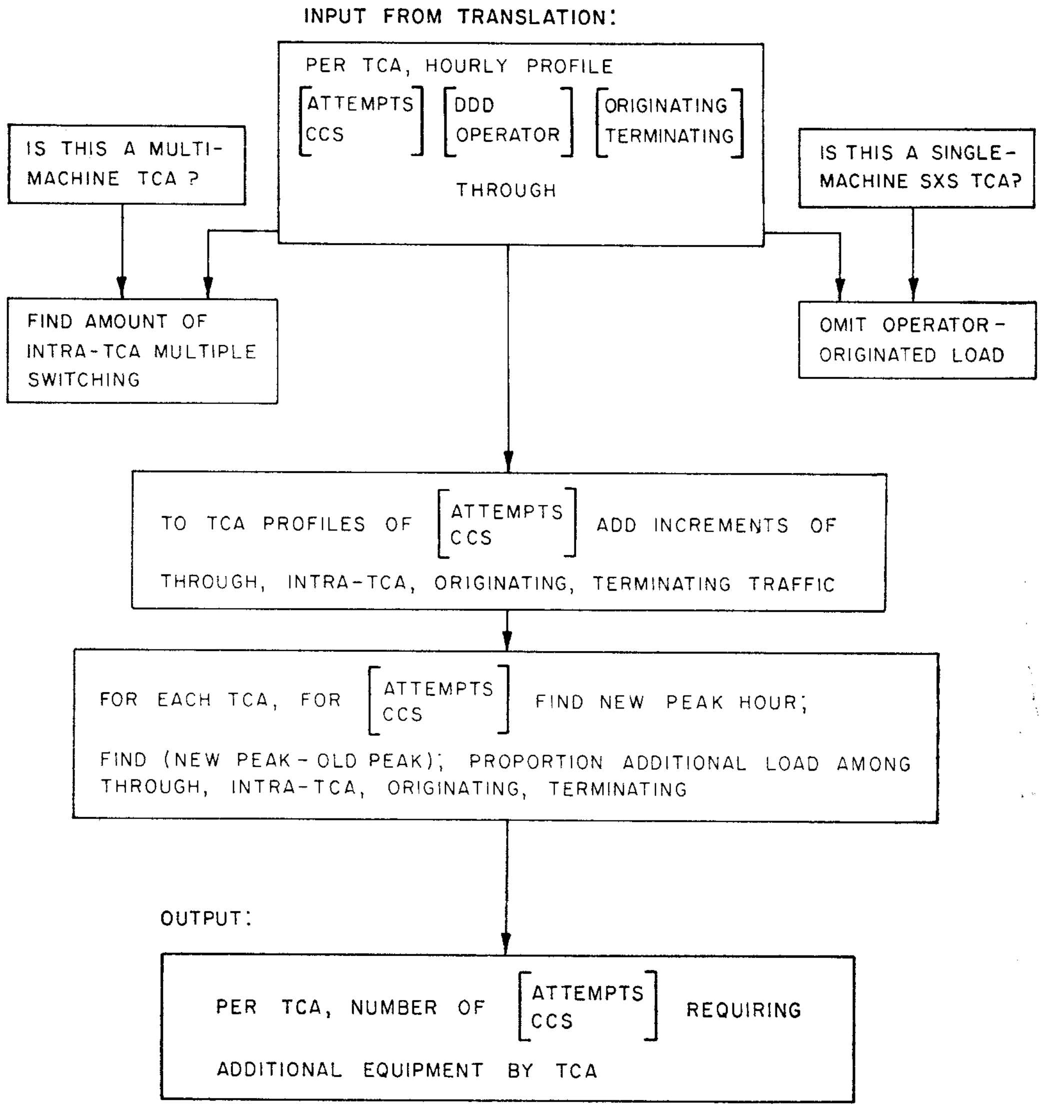
Note: At times, this factor will be negative. When originating traffic is routed directly from a local central office to a distant toll office, as between New Jersey central offices and New York City, the traffic associated with the originating equipment (New Jersey) is assigned a negative intra-TCA (double-switching) penalty.

Phase 1: New peak period requirements. In the module's first phase the effects on TCA peak periods of the incremental traffic are estimated; changes in the peak loads will alter equipment requirements. The estimates are made in a manner which reflects not only variations in the load characteristics of the switching machines, as observed in the base data, but also differences in the capacities of the various types of equipment in use.

The procedure is illustrated in the simplified block diagram shown in Figure 43. The output is expressed as incremental CCS and attempts requiring additional switching equipment.

FIGURE 43

SIMPLIFIED BLOCK DIAGRAM,
TOLL DIAL SWITCHING FACILITIES COST MODEL, PHASE I



Switching demand increments are input in the form of hourly profiles (generated by translation) of CCS and attempts for the total System by type of traffic — originating, terminating, and through; operator assisted and DDD. The analysis first develops disaggregate estimates of the 18-hourly-period incremental profiles for each switching machine center in the following manner:

$$IS(S)^{A_{s\tau oijn}} = \left[L(S)^{A_{s\tau oijn}} \left(\frac{B^{A_{s\tau oij.}} + \Delta B^{A_{s\tau oij.}}}{B^{A_{s\tau oij.}}} \right) - L^{A_{s\tau oijn}} \right],$$

where

$I(S)^A =$ estimated hourly incremental switching load,

$s =$ service: interstate MTS, intrastate MTS, interstate WATS, intrastate WATS, other,

$\tau =$ originating, terminating, through traffic,

$o =$ operator-assisted, DDD traffic,

$i =$ rate mileage steps (15),

$j =$ time periods (18),

$n =$ TCA number,

$A =$ attempts or CCS,

$B^A_{s\tau oij.} =$ estimated aggregate System loads for status (base).

$\Delta B^A_{s\tau oij.} =$ estimated incremental aggregate System loads (revised to reflect updates or effects of price change), and

$L_{s\tau oijn} =$ status switching hourly load profiles by TCA.

These incremental estimated loads are then used to adjust the status variables for each TCA and to analyze the effects of price changes.

The status variables are adjusted to reflect changes in time and in exogenous conditions (*i.e.*, changes in national economic variables associated with the demand models). The adjustment procedure is:

$$L'(S)^A_{s\tau oijn} = L(S)^A_{s\tau oijn} + I(S)^A_{s\tau oijn},$$

where

$$L'(S)^A_{s\tau oijn} = \text{ revised offered switching load.}$$

Then the total offered load for each switching location is computed:

$$L'(S)^A_{\dots jn} = \sum_s \sum_\tau \sum_o \sum_i \left[L'(S)^A_{s\tau oijn} \right].$$

The peak periods for attempts and CCS for both the updated base and the revised loads are computed by TCA:

New peak period $j'(n)$ = value of j for which $L'(S)^A_{\dots jn}$ is maximized.

Base or updated peak period $j^*(n)$ = value of j for which $L(S)^A_{\dots jn}$ is maximized.

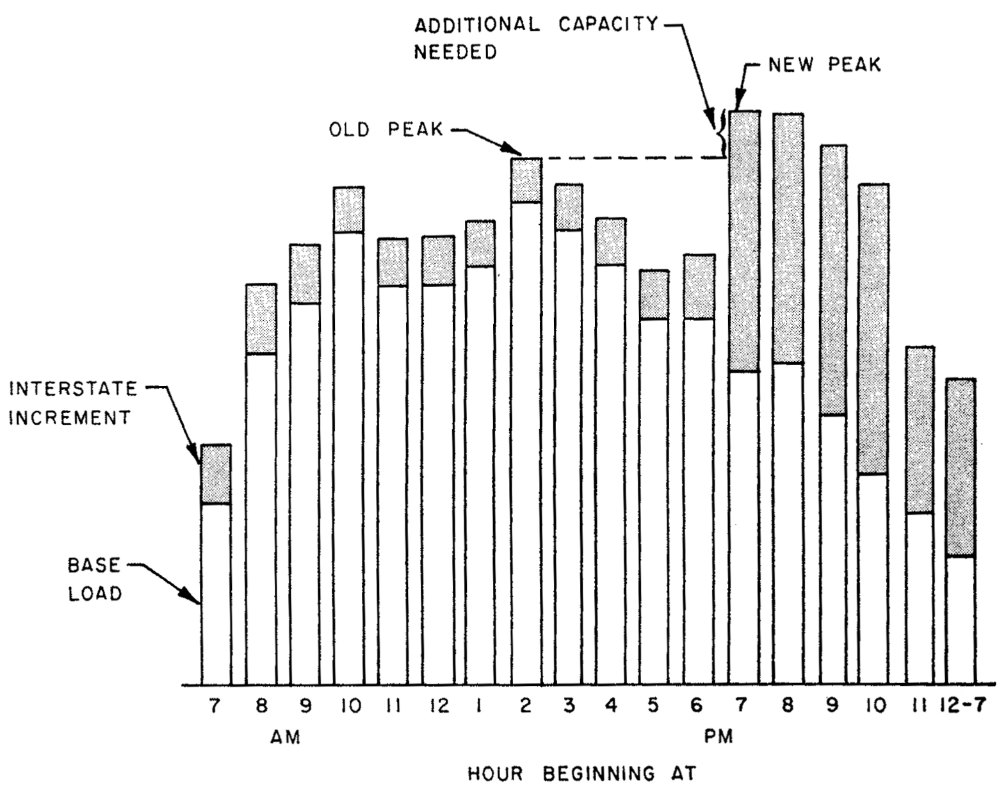
The change in peak period load is then given by:

$$\Delta L(S)^A_{\dots jn} = L'(S)^A_{\dots j'n} - L(S)^A_{\dots j^*n}.$$

Figure 44 illustrates how an increment of interstate traffic has changed the peak period and the requirement for additional capacity. Because the busy hour characteristics of

FIGURE 44

HOURLY PROFILE AFTER INTERSTATE INCREMENT HAS BEEN ADDED TO BASE – SEPARATE DISTRIBUTIONS FOR ATTEMPTS AND CCS



equipment sensitive to CCS and equipment sensitive to attempts are different, the changes in peak period and CCS requirements are computed separately.

The estimated new peak period requirements are then used to derive new estimates of the engineering loads in the TCAs:

$$L(S)^A_{\dots jn} = L'(S)^A_{\dots j'n} R_{\dots n} M(S)_{\dots n},$$

where

$R_{\ldots n}$ = ratio of ten-high-day (engineering) load to ten-day study period load for each TCA, and

$M(S)_{\ldots n}$ = multi-switching factor for toll center (n).

The incremental CCS and attempts that would be associated with a change in switching equipment requirements can now be computed:

$$\Delta L(S)^A = L(S)^A_{\ldots j'n} - L(S)^A_{\ldots jn} R_{\ldots n} M(S)_{\ldots n}$$

where

$\Delta L(S)^A$ = incremental change in CCS and attempts requiring new switching capacity.

This calculation is based on the assumption that the unused capacity at each switching center at the base period should be held more or less constant because it exists for a specific engineering or planning purpose. When growth and incremental traffic are added to the base profiles, this capacity is not changed.

The outputs of the first phase of the dial switching facilities investment sub-model are estimates of the CCS and attempts requiring additional switching equipment at the individual TCA peak periods resulting from a demand increment.

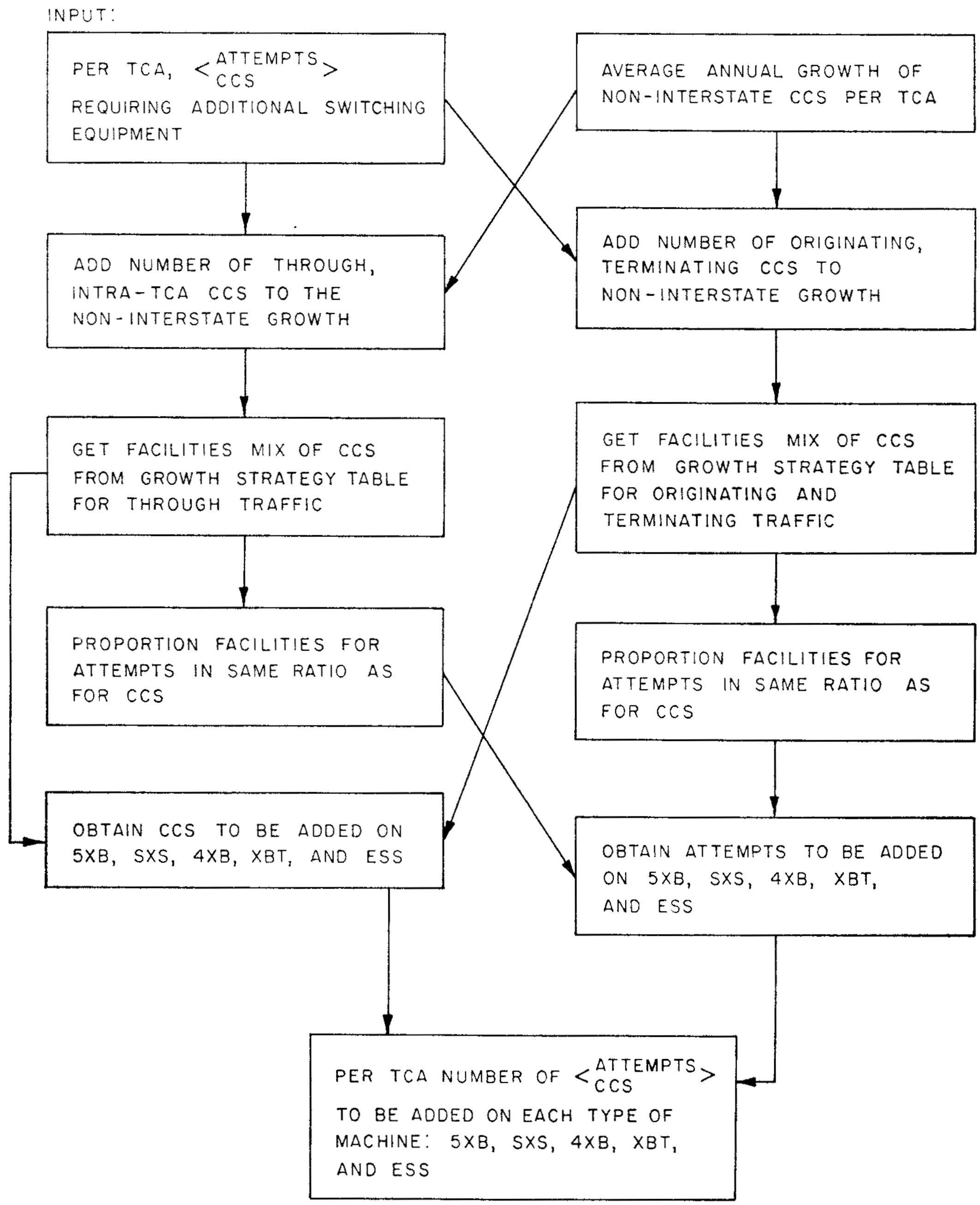
Phase 2: Facilities analysis. The objective of this phase of the sub-model is both to estimate the mix of incremental switching facilities that would minimize the investment required to serve incremental CCS and attempts at the peak periods and to reflect the economic timing of these investments. Figure 45 is a block diagram of the procedure, in which facility mix functions are applied. Other data required for this phase are growth estimates (derived from planning in formation), present switching capacity at each TCA, and the busy period engineered levels of traffic at each TCA.

The facility mix functions are based on statistical analysis of existing and planned installations, based in turn on data acquired from the Associated Companies and from Long Lines. They are expressed as probabilities of the apportionment of incremental peak-load traffic among the major types of equipment. Table 13 shows examples of these probabilities, which are in the form of a "strategy" table.

Total TCA growth plus increment comprise the CCS and attempts loads for which additional equipment is required, and indicate the row on the strategy table to be referenced for a given demand change. The table, showing typical probabilities of load distribution among four types of Bell System switching equipment — Nos. 4 and 5 Crossbar, Crossbar Tandem, and Step-by-Step — reflects the mix and timing of equipment investments planned by the Associated Companies and Long Lines.

FIGURE 45

TOLL DIAL SWITCHING FACILITIES COST MODEL, PHASE 2:
TIMING AND MIX OF SWITCHING FACILITIES



The model computes

$$[I(S)^{A}_{ent}],$$

where

$$\begin{aligned} A &= \text{CCS or Attempts,} \\ n &= \text{TCA index,} \\ e &= \text{equipment type index,} \\ t &= \text{timing of investment at switching center } n\text{, and} \\ I(S) &= \text{incremented load,} \end{aligned}$$

to determine the additional equipment required to minimize capital investment.

The Phase 2 results are the number of incremental CCS and attempts served by each equipment type, and the timing of future investments required to serve the new demand.

TABLE 13

SAMPLE OF BELL SYSTEM STRATEGY TABLE FOR ALLOCATING PEAK LOAD EQUIPMENT REQUIREMENTS OVER PRESENT SWITCHING TECHNOLOGIES (NOT INCLUDING ELECTRONIC SWITCHING SYSTEM)

	LOWER BOUNDARY OF GROWTH CATEGORY (CCS)	PROBABILITIES (%) SXS	5XB	XBT	4XB	
1	$-\infty$	.140	.860	.0	.0	
2	1	.734	.266	.0	.0	
3	51	.722	.278	.0	.0	
4	101	.525	.475	.0	.0	
5	151	.451	.549	.0	.0	
6	201	.513	.487	.0	.0	
7	251	.405	.595	.0	.0	
8	351	.406	.551	.043	.0	
9	451	.491	.479	.030	.0	
10	651	.483	.345	.173	.0	*
11	1001	.180	.322	.498	.0	
12	2001	.038	.200	.693	.069	
13	4001	.004	.002	.445	.553	*
14	6001	.003	.005	.271	.721	
15	9001	.002	.001	.063	.935	*
16	15001	.0	.001	.082	.917	
17	20001	.0	.0	.150	.850	
18	30001	.0	.0	.202	.798	
19	40001	.0	.0	.097	.903	
20	60001 (+ ∞)	.0	.0	.042	.958	

*DUE TO ROUND-OFF, THESE ROWS DO NOT TOTAL 100%

Phase 3: Book costs. The costs of the additional equipment are computed in the last phase, based on the equipment mix derived in Phase 2. Cost functions are used, based on

engineering data for new starts (that is, new switching machines) and additions (that is, additions to existing machines) over a given period of time, comprising:

(a) Number of CCS added,
(b) Number of attempts added,
(c) Total number of frames added,
(d) Gross additions,
(e) Western Electric engineering cost,
(f) Western Electric material cost,
(g) Western Electric installation cost, and
(h) Telephone company expense.

The frames for the cost functions are classified as attempts-sensitive, CCS-sensitive, AMA recording equipment-sensitive, or required for loading (traffic and test frames).

For each machine type (e) and type of addition (k, new start or addition), the following factors are developed:

f_{1ek} = number of attempts-sensitive frames added for each incremental unit of attempt capacity, type k,
f_{2ek} = number of CCS-sensitive frames added for each incremental unit of CCS capacity, type k,
c_{1ek} = cost of adding attempts-sensitive frames for each incremental unit of attempt capacity, type k, and
c_{2ek} = cost of adding CCS-sensitive frames for each incremental unit of CCS capacity, type k.

First, for each (e) and (k) the sum of the (i) frames (attempts-, CCS-, loading-, or AMA-sensitive) is determined:

$$n_{iek} = I(s)^{A}_{ent}/\alpha_{iek},$$

where

$$\alpha_{iek} = \text{capacity } (i) \text{ per frame for each } e \text{ and } k.$$

Then the sum of the (i) capacity for each (e) and (k) is computed (s_{iek}). The total cost for each (e) and (k) is DOL_{ek}, and the total frames are TFA_{ek}.

A loading factor, l_{iek}, is then computed to derive the number of loading frames based on the number of attempts-sensitive and CCS-sensitive frames (since loading relates to both

CCS and attempts):

$$l_{iek} = \frac{\eta_{iek}}{\eta_{1ek1} + \eta_{n_2ek1}}(TFA_{ek} - \eta_{1ek} - \eta_{2ek} - \eta_{4ek}),$$

where

$$\begin{aligned} n_{iek} = {} & \text{ number of frames,} \\ i = {} & 1, \text{ attempts-sensitive frames,} \\ = {} & 2, \text{ CCS-sensitive frames,} \\ = {} & 3, \text{ loading frames, and} \\ = {} & 4, \text{ AMA-sensitive frames.} \end{aligned}$$

The quantities of attempts- and CCS-sensitive frames added to serve an incremental unit of attempt or CCS capacity are then computed:

$$f_{iek} = l_{iek} * n_{iek}, \; i = 1, 2.$$

The total cost of adding attempts-sensitive and CCS-sensitive frames for each incremental unit of capacity is then computed as:

$$c_{iek} = f \frac{DOL_{ek}}{TFA_{ek}} \; i = 1, 2.$$

The total number of attempts- and CCS-sensitive frames and their total costs are computed, based on these factors applied to the peak period CCS and attempts by type of equipment (e) and by year as follows:

$$F_{.} = \sum_t F_t = \overbrace{\sum_{e=1}^{4} \left[\overbrace{\sum_{i=1}^{2} \left(\overbrace{\sum_{k=1}^{2} w_{iek} f_{iek}}^{\text{STEP 1}} \right) S_{ie}}^{\text{STEP 2}} \right]}^{\text{STEP 3}}$$

$$I_t = \overbrace{\sum_{e=1}^{4} \left[\overbrace{\sum_{i=1}^{2} \left(\overbrace{\sum_{k=1}^{2} w_{iek} c_{iek}}^{\text{STEP 1}} \right) S_{ie}}^{\text{STEP 2}} \right]}^{\text{STEP 3}},$$

where

$$
\begin{aligned}
F_t &= \text{Frames for year } t,\\
I_t &= \text{investment cost for year } t,\\
i &= 1,\ \text{attempts-sensitive},\\
&= 2,\ \text{CCS-sensitive},\\
e &= 1,\ \text{Crossbar Tandem equipment},\\
&= 2,\ \text{Step-by-Step equipment},\\
&= 3,\ \text{No.4 Crossbar equipment},\\
&= 4,\ \text{No 5 Crossbar equpment},\\
k &= 1,\ \text{new starts},\\
&= 2,\ \text{additions},\\
w_{iek} &= \text{weights relating the number of new starts and the number of}\\
&\quad\ \text{additions such that for the eth machine type } (\textstyle\sum_{k=1}^{2} w_{iek} = 1),\\
S_{ie} &= \text{number of attempts (capacity) or CCS handled on } e\text{th machine.}
\end{aligned}
$$

In Step 1, the quantity of (attempts, CCS)-sensitive frames and the cost of the frames are computed for the estimates of required incremental units of capacity, for each of the four equipment types. In Step 2, this quantity and cost are multiplied by the capacity required to serve the demand model forecast increments, and summed to derive total frames and costs by equipment type. In Step 3, the model sums over all equipment types, deriving total frames and costs for the Bell System.

The costs of building and land associated with toll switching equipment are determined as functions of the number of frames of each type of equipment that is required to serve the incremental demand. The cost factors are combinations of square feet per frame and costs per square foot, and are applied to the total number of frames per equipment type. These results are then summed over all types of machine, producing Bell System total incremental toll switching costs for land and buildings by year of investment. The Bell System Standard Accounts applicable to these investment categories are:

Account 221 — central office switching equipment by major class,
Account 211 — land, and
Account 212 — buildings.

Module 7: Special switching equipment requirements (AMA). This part of the analysis estimates the additional automatic message accounting equipment (AMA) required to serve the estimated incremental traffic forecasted by the demand models. AMA equipment consists of four basic parts:

(1) Paper tape perforators,

(2) Recorders, which operate the perforators,

(3) Transverters, which translate the dialed digits into pulses to activate the recorder, and

(4) Line identity equipment, which identifies the calling party.

These four elements are interrelated at the switching location by connectors, and are installed according to certain required mixes. For example, usually one perforator and one recorder are required for approximately 90 trunks, while only a fraction of the other pieces is needed.

Data. The variables of interest are:

(1) *Endogenous variables*

These comprise the special switching equipment requirements, and the incremental special switching equipment capital investment (long-run incremental costs).

(2) *Exogenous variables*

The input data are both module-enogenous (*i.e.*, developed within the model, in another module) and model-exogenous. The module-enogenous data are —

A. Incremental peak switching loads $\left[\hat{L}(S)\right]$, derived from the switching module by TCA, and

B. Incremental toll connect circuits by TCA $[C(TC)]$.

The model-exogenous data are —

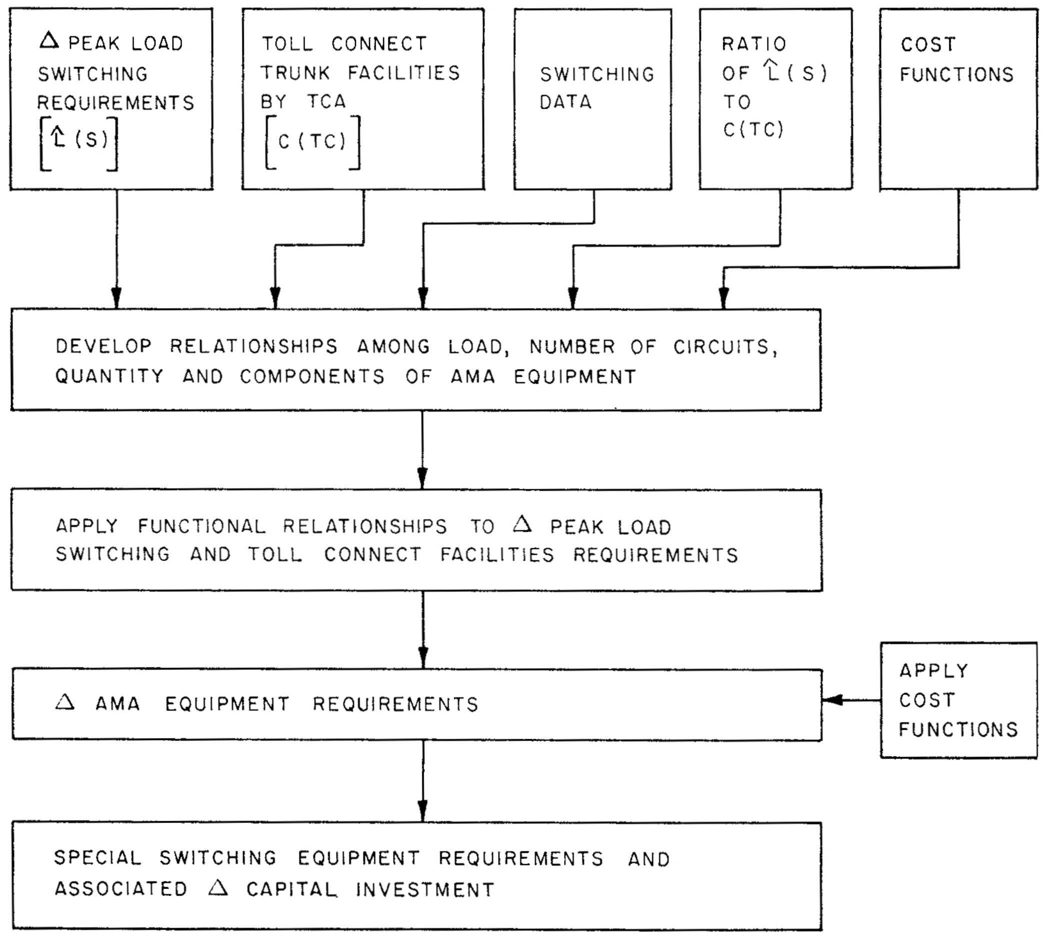
A. The functional relationship $f\left[\hat{L}(S), C(TC)\right]$ used to estimate the AMA equipment requirements, and

B. Cost functions for the four basic equipment components.

Procedure. Figure 46 is a block diagram of the procedure used. AMA equipment requirements are estimated as a function of both the incremental peak load (derived from the switching facilities module) and the number of incremental circuits (derived from the toll connect facilities module) by TCA. Available historic Associated Company switching machine data were analyzed to develop relationships among load, number of circuits, and quantity and components of AMA equipment. These relationships are the functions which are applied to the incremental load and circuit data derived from the other modules in order to estimate additional AMA equipment requirements.

FIGURE 46

AMA SWITCHING EQUIPMENT REQUIREMENTS, SIMPLIFIED BLOCK DIAGRAM

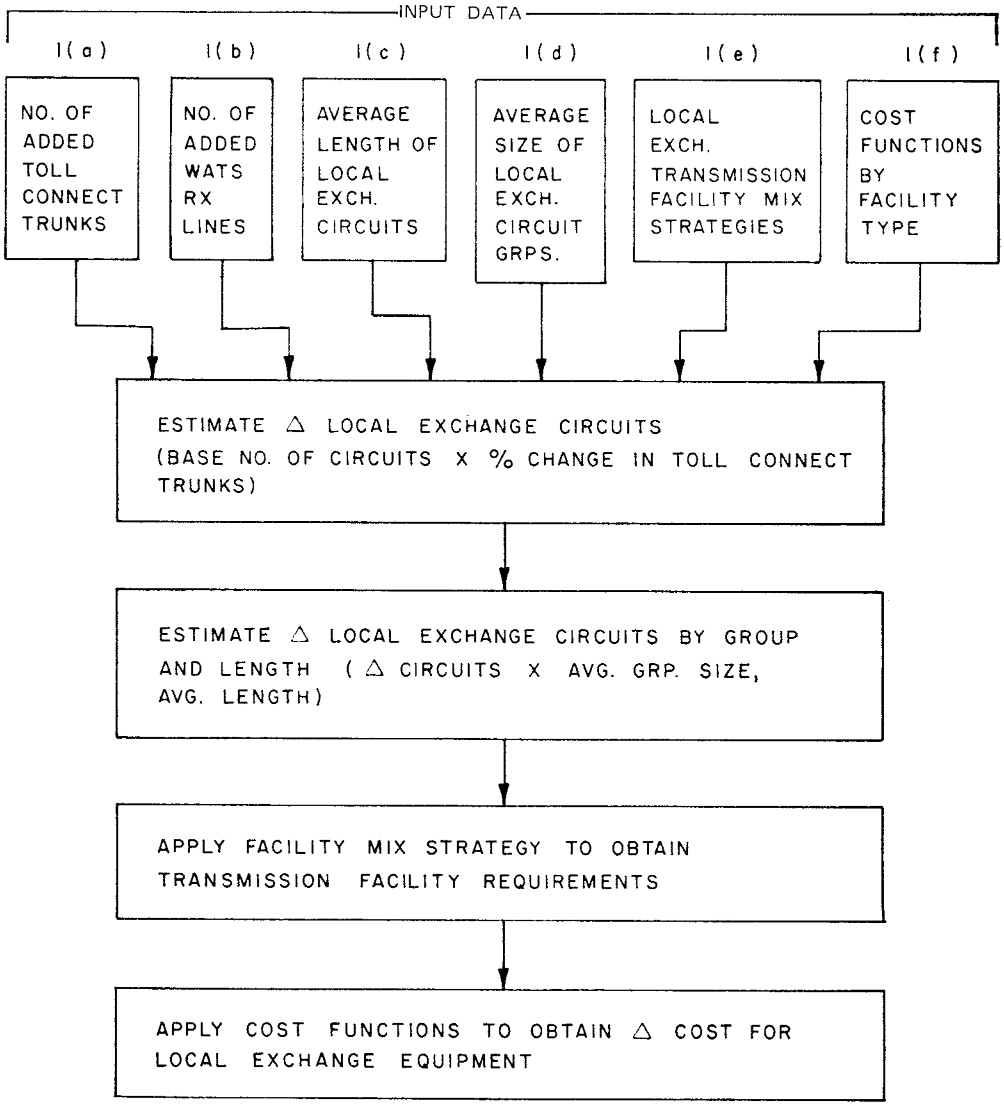


Average cost functions, also developed from historic Associated Company data, are applied to these equipment estimates in order to estimate the incremental AMA investment costs.

Module 8: Local exchange plant capital investment. Local switching equipment, local "loops" (circuits connecting customer equipment to the central office), and station equipment (customer telephones, etc.) are considered relatively insensitive to changes in the demand for toll telephone service. However, two categories of local plant are exceptions: Circuits within the exchange area which serve proportionately large amounts of toll traffic, and WATS exchange lines which connect subscribers to the offices in which billing information is recorded. These two types of local circuit are affected by toll demand changes, and are therefore included in this analysis. Figure 47 is a block diagram of the procedure followed.

FIGURE 47

BLOCK DIAGRAM,
LOCAL EXCHANGE PLANT CAPITAL INVESTMENT



Data. The following are the categories and types of variables relevant to procedures within this module:

(1) *Endogenous variables*

These comprise data on the exchange plant facilities associated with the two categories of circuit mentioned above, namely,

A. Incremental exchange transmission circuits,

B. Capital investment associated with (A),

C. Incremental WATS exchange lines, and

D. Capital investment associated with (C).

(2) *Exogenous variables*

The six categories of input data required are —

A. Number of toll connect trunks to be added (derived from module 5),

B. Number of WATS RX lines to be added (derived from module 2),

C. Average length of local exchange circuits,

D. Average size of local exchange circuit groups,

E. Local exchange transmission facility mix strategy table, and

F. Cost functions by facility type.

Procedure. The local exchange circuits are usually short, on average about 8 miles long. A study was undertaken to develop statistics on the average size of the circuit groups, their average lengths of haul, and the number of circuits in each exchange area. Statistical analysis showed that increases in the number of short-haul toll connect trunk groups correlated with increases in the local exchange circuits. It was then possible to estimate the number of incremental local exchange circuits by multiplying the existing number by the percentage in crease in toll connect trunks (obtained from the fifth module's output).

The incremental number of local circuits are then multiplied by the average circuit group size and by the average length of haul, in order to obtain estimates of the incremental circuit requirements by groups and by lengths. Facility mix functions are applied to estimate the transmission facilities requirements, and finally cost functions are used to estimate investment costs. (The cost functions were developed on principles similar to those described in module four).

For the WATS exchange lines, the estimated number of incremental WATS lines produced by module 2 is used as a basis for estimating incremental investment associated with local circuits. An average incremental cost per local line is multiplied by the incremental WATS lines. Since this capital investment is required for the test year, time (t_0), the average costs are assumed to give a reasonable approximation for average long-run incremental costs. Because further changes in the local plant facilities are regarded as second-order effects with respect to a change in the demand for toll telephone service, they were omitted from this initial version of the model. However, investigations into other areas of the local exchange are planned for the future as the model is refined.

Module 9: Other capital investments. Three other categories of capital investment are included in the model:

(1) Building investment associated with general office space (Bell System Account 212),

(2) Furniture and office equipment (Bell System Account 261), and

(3) Vehicles and other work equipment (Bell System Account 264).

Although the effects on these are small compared to the effects on major capital investments, they must nevertheless be included in estimates of the long-run incremental cost incurred in serving increments of toll telephone demand. Statistical studies were undertaken to derive estimates of the relationships of incremental changes in the above categories to specific rate base changes over a given period of time. These estimated ratios were then applied to the incremental plant investment to determine effects on other capital categories of investment.

Data. The following are the endogenous and exogenous variables of interest:

(1) *Endogenous variables*

A. Incremental investments associated with
   Δ General office space (part of Account 212),
   Δ Furniture and office equipment (Account 261), and
   Δ Vehicles and other work equipment (Account 264).
B. Incremental telephone plant under construction (Account 100.2).
C. Incremental property held for future telephone use (Account 100.3).
D. Incremental investment associated with material and supplies (Account 122).

(2) Exogenous variables

A. Module-exogenous data —
   1. Incremental investment in Account 100.1 (telephone plant in service).
   2. Incremental investment in the ith sub-account of Ac count 100.1.
B. Model-exogenous data —
   1. Ratios of incremental investment in general office space, furniture and office equipment, and vehicles and other work equipment to Account 100.1 (estimated).

Building Investment (Account 212). In Account 212 are reported the various investments associated with telephone company buildings, including the costs of all permanent fixtures, machinery, appurtenances, and appliances installed on company premises. Within the model, most of this investment category is included in estimates of the major capital account increments. However, that portion of building investment associated with general office space was estimated separately. A ratio of increments in investment associated with general office space to increments in total plant investment was estimated by studying data for a five-year period. This ratio is applied to the estimated increments in Account 100.1, which represent the change in telephone plant investment. The procedure is:

$$\Delta I(A)_{212} = R(O)_{212} * \Delta I_{100.1},$$

where

$\Delta I(A)_{212}$ = Increment in general office space investment,
$\Delta I_{100.1}$ = Incremental investment in Account 100.1 developed by summing the incremental capital investments (land, buildings, circuit equipment, switching equipment, exchange plant) derived in earlier modules, and
$R(O)_{212}$ = Ratio of incremental investment for general office space to Account 100.1 (excluding Accounts 261 and 264) for a five-year period.

Furniture and Office Equipment (Account 261). The capital investment reported in this account is associated with the acquisition of furniture and equipment for offices, storerooms, shops, and other company quarters not accounted for elsewhere. Bookcases, cabinets and filing cases, chairs, clocks, desks, and equipment in company restaurants and dining rooms are typical Account 261 items. Procedures similar to those used in estimating the general office expenses re used here. Thus,

$$\Delta I(A)_{261} = R(0)_{261} * \Delta I_{100.1},$$

where

$\Delta I(A)_{261}$ = Incremental investment in furniture and office equipment,
$\Delta I_{100.1}$ = Incremental investment in Account 100.1, and
$R(O)_{261}$ = Ratio of incremental investment in furniture and office equipment to increments in Account 100.1 over a five-year period.

Vehicles and Other Work Equipment (Account 264). Items reported in Account 264 include the cost of vehicles, tools, garage and shop machinery and equipment, and miscellaneous work equipment not included in other accounts. The procedure for estimating changes in this investment category is similar to those described above. Therefore,

$$\Delta I(A)_{244} = R(O)_{264} * \Delta I_{100.1},$$

where

$\Delta I(A)_{264}$ = Incremental investment in vehicles and other work equipment,
$\Delta I(A)_{1000.1}$ = Incremental investment in Account 100.1, and
$R(O)_{264}$ = Ratio of incremental investment in vehicles and other work equipment to increments in Account 100.1 (excluding Accounts 261 and 264) over a five-year period.

Module 10: Traffic operator board investment. Incremental traffic board investment related to increments in toll traffic is estimated in a two-phased procedure. The first phase estimates operator positions required as a result of a structured demand increment, and the second estimates the investment costs of facilities requirements. Figure 48 diagrams the procedure.

Data. The data are:

(1) *Endogenous variables*

These include incremental traffic operator board equipment requirements and incremental traffic operator board capital investment, both by type of equipment.

(2) *Exogenous variables*

The module-exogenous variables are the number of new positions required at individual chief operator units to serve incremental peak traffic both at toll boards and in directory assistance offices. The model-exogenous data include a facility mix strategy table, cost functions for equipment included in the planning strategies, growth factors based on System planning investment analysis, and the ratio of directory assistance equipment (positions) to directory assistance work units.

(3) *Status variables* These include information on the planning areas for switching, such as the present capacity of existing offices, the types of equipment which they contain, and the level of work load at the busy period (engineered level).

Phase 1: Incremental positions. Incremental positions are estimated separately for toll boards and directory assistance. For toll, the study period used is the same as that used in computing operator expense and work units (a procedure described in the next module). In Module 11, the number of additional operators, by hour of day, is used to estimate the required additional positions. For each individual Bell System switchboard, the number of new positions is estimated as

$$\Delta P_i = MAX_j\left[\Delta\sigma_j\right],$$

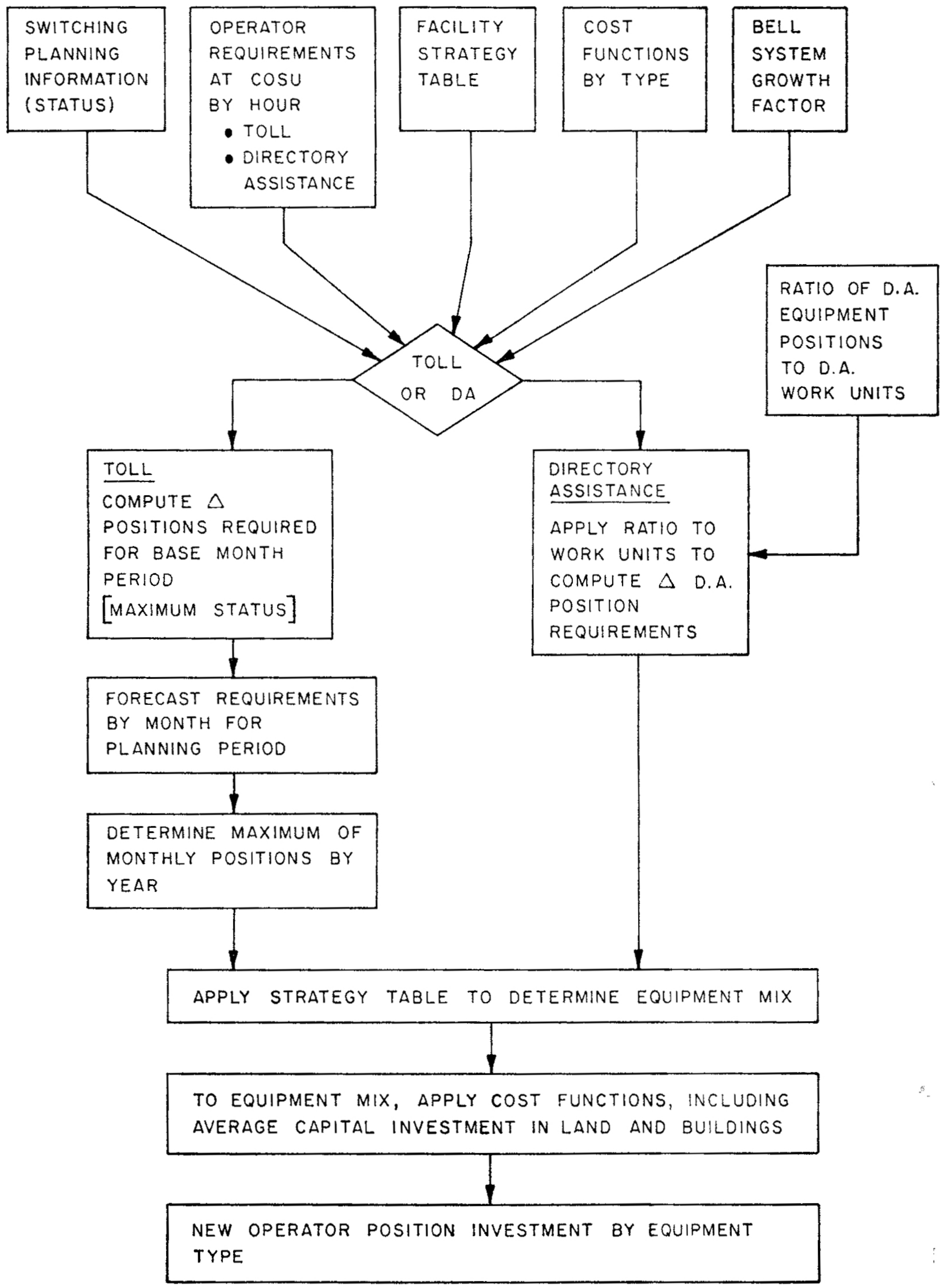
where

$\Delta P_i =$ number of new positions,
$i =$ chief operator unit, and
$\Delta\sigma_j =$ additional operators required during period j.

The number of directory assistance positions is determined by referencing the incremental directory assistance work units which are part of the output of the aggregate model used in estimating traffic office expense. The ratio of directory assistance positions to work units is presumed constant for all increments in demand. Therefore the present ratio is applied to the incremental work units, so that

FIGURE 48

TRAFFIC OPERATOR BOARD INVESTMENT, SIMPLIFIED BLOCK DIAGRAM



$$\Delta P_{DA} = W_{DA} * \tau_{DA}$$

where

W_{DA} = incremental work units for the total Bell System,
τ_{DA} = ratio of directory assistance positions to work units, and
ΔP_{DA} = the number of incremental positions required to give the desired grade of service.

Phase 2: Facilities mix and capital investment. For the manual toll operator switching equipment, facility mix strategy tables similar to that used for the dial switching equipment (Module 6) are referenced. The inputs for estimating facility mix and capital investment are the number of operator positions, by type of office and by office code; status variables reflecting the present engineered capacity of each office, its type of equipment, and the present level of work units during the office busy period; growth characteristics derived from AT&T's investment planning program for each office; and the mix strategy tables, separate for directory assistance and toll.

The facility mix tables comprise percentage mixes of equipment which are probabilities reflecting future apportionments of work unit loads over types of equipment, as a function of the size of operator position increment. These are average Bell System facilities strategies, and are also used to estimate investment costs as a function of office size and increment. From this information, the timing of new investments is also known, which makes it possible to estimate dates at which the incremental positions would be required.

The incremental cost functions used for the major types of manual switching equipment, both toll and directory assistance, were derived from the Associated Companies and from the Long Lines construction program. Added to these costs are average capital investments in land and buildings estimated on the basis of an average square foot age of office space per position. The costing procedure is of the form

$$C_{\mu t.} = \sum_i P_{\mu ti} D_{\mu t},$$

where

μ = type board,
t = time of investment,
$P_{\mu ti}$ = estimated incremental positions, weighted for relative work efficiency,
ii = chief operator unit for toll operator positions,
$D_{\mu t}$ = investment cost per board determined from facility mix tables, and
$C_{\mu t.}$ = operator position investment by type.

Module 11 : Traffic expense. A major expense associated with furnishing toll service is the payment of operator wages and of the costs of traffic supervision, employee training, and lunch room operations. Some traffic engineering costs not associated with construction

are also included in the general category labeled "traffic expense." The following accounting categories and related expenses are included in Module 11:

(1) *Operator Wages (Account 624).* This includes the salaries of chief operators, assistant chief operators, supervisors, switch board operators, information operators, directory assistance operators, and so on-all personnel who help subscribers place and complete calls, as well as recording incidental complaints. Account 624 also contains reports on the pay of clerks and others engaged in line assignments, taking message register readings, preparing time or attendance records, and the like.

(2) *General Traffic Supervision Expenses (Account 621).* These include salaries paid to supervisors directly in charge of the Traffic Offices, as well as their office, travel, and other miscellaneous expenses.

(3) *Rest and Lunch Room Expenses (Account 626).* These are the costs incurred in operating rest and lunch rooms for Traffic Office employees.

(4) *Operators' Employment and Training Expenses (Account 627).* These are incurred in the employment and training of operators.

(5) *Miscellaneous Traffic Expenses (Accounts 629, 630, 631, and 633).* These are miscellaneous expenses for postage, printing and stationery, house services, lighting, heat, janitor services, and other categories not chargeable to other accounts.

Operator Wages (Account 624). The annual incremental traffic expense related to payment of operator wages incurred as a result of handling increments of toll traffic is estimated in this module. Both interstate and intrastate demand changes may be considered in this procedure, but for simplicity the description will focus on interstate toll usage changes. The additional work units required per chief operator switching unit (COSU) at different time periods as a result of a structured increment in demand are estimated first. Then estimates of the number of incremental operators needed to provide these work units are developed. Finally, the cost of those operators is estimated. The cost of other necessary traffic personnel not at the boards is also computed. Figures 49 and 50 illustrate the procedure.

Data. The data are:

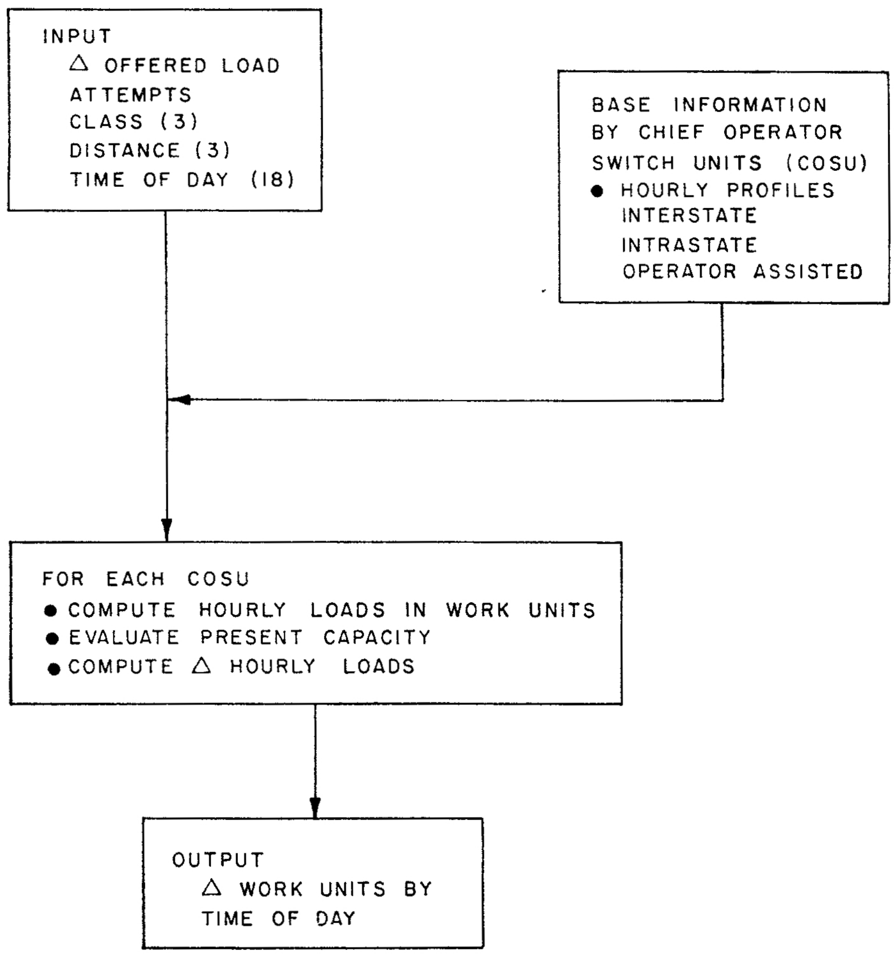
(1) *Endogenous variables*

These include incremental work units by time period, by COSU for ABD (average business day); incremental operators by time period, by COSU for ABD; and annual traffic expenses.

FIGURE 49

TRAFFIC EXPENSE ANALYSIS:

EVALUATION OF HOURLY LOAD REQUIREMENTS FOR 3CL AND TSPS OPERATOR BOARDS

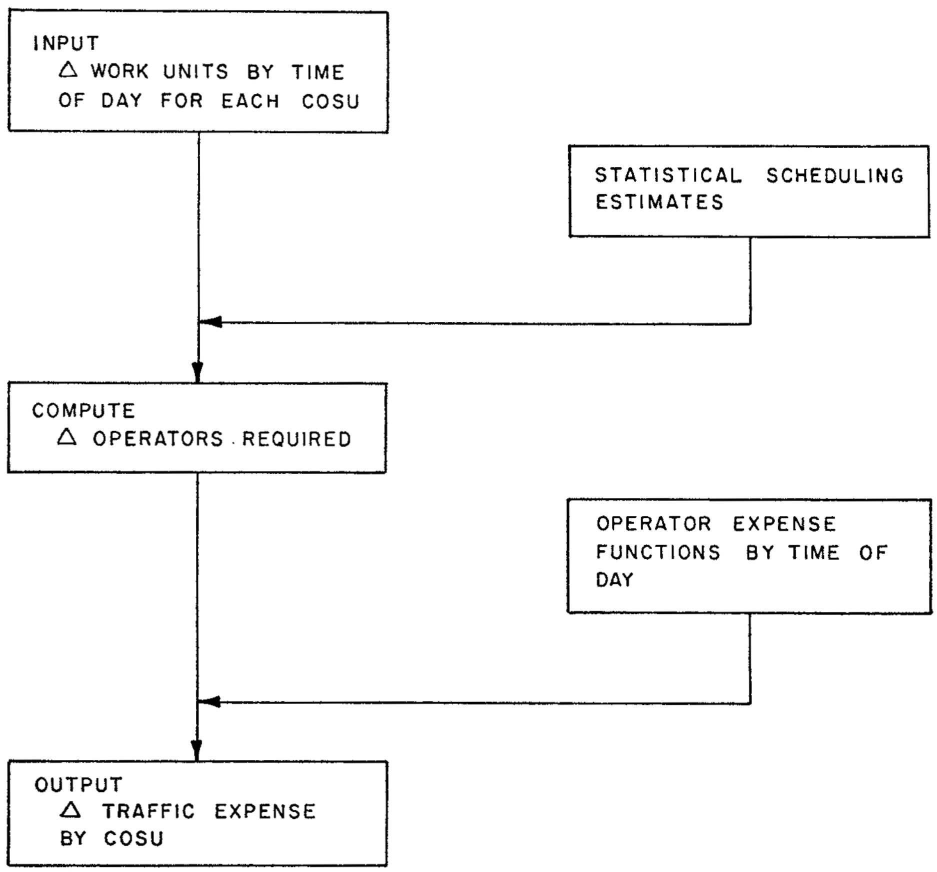


(2) *Exogenous variables*

These are both module-exogenous and external. The module-exogenous data represent the incremental traffic office loads by 3 classes of call, 15 distance categories, and 18 local times of day. The external variables are the statistical operator schedules, work unit coefficients (by type of call), coefficients to reflect weekend and average business day levels, coefficients to reflect seasonal levels by COSU and by type of office, and a directory of Traffic Offices.

FIGURE 50
TRAFFIC EXPENSE ANALYSIS:

OPERATOR AND EXPENSE ANALYSIS



(3) *Status variables*

Representing the pre-incremental status, the module uses for the hourly profiles of operator work units by COSU (interstate, intrastate, and operator assistance), estimated operators on duty at each COSU, number of positions at each COSU, and statistics on the level of directory assistance toll calls presently being processed in the System.

Determination of operator work units. The number of operator work units involved in serving the demand occurring at all Bell System COSUs for 18 periods of the day are estimated for the "base" demand and then revised to reflect the effects of demand change. The base and incremental demand were developed earlier, in the translation (Module 3), and are given as the number of toll calls of a given type which fall into a specified category,

or cell. Each type of cell is described by a combination of the following factors:

(a) Call classification
- — operator person-to-person
- — operator station-to-station

(b) Calling station
- — non-coin
- — coin

(c) Distance (length of haul)
- — short
- — medium
- — long

(d) Time of day
- — day
- — evening

[Note that in this module the distance (15) bands are reduced to 3 classifications and the times of day (18) to 2 classifications. This is because of the structure of operator work coefficients (that is, operator work units associated with classes of calls) that were available.]

There are therefore 24 possible types of call that can be incremented on a percentage basis. Status data on the number of work units actually carried at each COSU are available for the following combinations of parameters:

(a) Time of day
- — 8 time intervals, one hour or longer

(b) Day of week
- — average business day (ABD)
- — weekend day (WED).

The type of switchboard (cord, TSPS,[21] or directory assistance) is nested within the identification of the COSU, of which there are about 1600 in the Bell System. There are three distinct estimating procedures, or sub-models: one for cardboards, one for TSPS, and one for directory assistance. Each of these is further subdivided into an average business day (ABD) and a weekend day (WED) model.

The cordboard model. The following definitions are used in this description:

$i =$ type of ticketed toll call (i = 1, 2,$\cdots$, 24),
$j =$ time period of the day (j = 1, 2,$\cdots$, 18),
$d_{ij} =$ number of ith type calls to be handled during jth period,
$u_{ij} =$ work-unit coefficient, including an adjustment for "additives,"[22] for a call in cate-

[21] TSPS: Traffic Service Position System.

[22] "Additives" are a means of weighting work-unit coefficients to reflect the complexi-

gory ij,
$t_j =$ coefficient giving toll assist work units per toll call during jth period,
$\eta_j =$ number of ith type calls to be handled during jth period,
$AS_j =$ total number of toll assist work units in period j.

In the notation used, a single prime after a variable will designate its value prior to a demand change. A double prime will designate its value after a demand change. The following assumptions are made :

(A1) The work-unit coefficients u_{ij} and the local assistance load L_j are not affected by the rate change.
(A2) The toll assist work units AS_j and toll assist work-unit coefficients t_j are not affected by the rate change.

In Assumption (A2), the effect of the rate change on intrastate calls and assistance is neglected because:

(a) Currently there is no method (nor are there data) for estimating the effect,
(b) The effect probably depends upon rates in a very complex way, and
(c) Cross-elasticities are extremely difficult to measure.

The effect of the rate change on toll assist work units per toll call is also neglected in Assumption (A2) because it is thought to be very small and would be very difficult to measure. The model estimates the number of operator toll work units prior to a demand change, W'_J, as

$$W'_j = \sum_i (u_{ij} + t_j) d'_{ij}.$$

W'_j can also be obtained as data.

This calculation, together with Assumptions (A1) and (A2), leads to the following:

$$\begin{aligned} W''_j &= \sum_i (u_{ij} + t_j) d''_{ij}. \\ &= \sum_i (u_{ij} + t_j) d'_{ij} + \sum_i (u_{ij} + t_j) m_{ij}, \\ &= W'_j + \sum_i (u_{ij} + t_j) m_{ij}, \end{aligned}$$

where

d'_{ij} and $d''_{ij} =$ number of type ij calls before and after a demand change, and
$m_{ij} =$ the estimated increment in the number of type ij calls.

Now a term π_j will be defined as the proportional increase in W'_j as a result of the demand change. Then

$$\pi_j = \sum_{i=1}^{12} m_{ij} w_{ij} \bigg/ \sum_{i=1}^{12} d_{ij} w_{ij}, \quad j = 1, 2, \cdots, 10,$$

---

ties involved in serving certain types of operator-handled calls.

and

$$\pi_j = \sum_{i=13}^{24} m_{ij} w_{ij} \Bigg/ \sum_{i=13}^{24} d_{ij} w_{ij}, \quad j = 11, \cdots, 18,$$

where

$d_{ij} =$ estimated number of type ij calls,
$w_{ij} =$ station coefficients based on the average number of work units per ABD station and person call,[23]

and the 12 types of daytime call are subscripted 1 through 12, and evening calls from 13 through 24.

One pair of matrices (d_{ij}) and (m_{ij}) is used for the whole Bell System. This is possible because these numbers are used on a scale-free basis; the calculations would not be affected if both matrices were multiplied by the same factor to scale them down to a size appropriate for an individual switchboard. The model can easily be adjusted if future demand translators provide custom-made matrices for each board. (The numbers d_{ij} and m_{ij} are not used for any other purpose than the calculation of π_j)

Determination of incremental operator requirements. Each COSU is first adjusted to reflect to some extent the relative operating characteristics. The total number of work units handled in a day are divided by the total number of operator hours available to handle that load. Thus, for a single day,

$$BL = \sum_{j=1}^{18} V_j \Bigg/ \sum_{j=1}^{18} \sigma_j,$$

where

$BL =$ board load,
$V_j =$ volume of work units to be handled in period j, and
$\sigma_i =$ number of available operator hours in period j.

[23] Here, w_{ij} has one constant value for given i over $j = 1$ through 10 (daytime), and a second constant value for given i over $j = 11$ through 18 (evening).

Table 14 is a section taken from a force management table used for staffing traffic offices in

TABLE 14

EXTRACT FROM FORCE MANAGEMENT TABLE

WORK UNITS PER HOUR	NUMBER AT THE BOARD	BOARD LOAD
146	2	73
305	3	102
493	4	123
694	5	139
1,135	7	162
1,845	10	185
3,108	15	207
4,423	20	221
5,783	25	231
7,150	30	238
8,548	35	244
9,907	40	248
12,475	50	249
15,055	60	251
17,646	70	252
20,326	80	254

the Bell System. Let σ_j' be the number of operator hours required to handle the actual carried load V_j', for $j = 1, \cdots, 17$; for $j = 18, \cdots, 24$, let σ_j' be operator hours required to handle the average carried load V_{18}'. Let the theoretical daily total be $\sigma' = \sum_{j=1}^{18} \sigma_j'$ operator hours. Then the theoretical board load BL' is

$$BL' = \sum_{j=1}^{18} V_j'/\sigma'.$$

This may now be compared with the actual board load BL. If $BL' > BL$, it will be worthwhile to reduce the coefficients used in deriving V_j', on the basis that the switchboard is actually taking less than the average Bell System time to handle toll calls. Similarly, if $BL' < BL$, the coefficients should be increased. Changing the value of the coefficients will cause a corresponding change in V_j'. The adjusted work units in period j are

$$V_j^* = V_j' \frac{BL}{BL'}$$

before the rate change. Similarly,

$$V_j^{**} = V_j'' \frac{BL}{BL'}$$

is the adjusted load after the rate change. V_j^* and V_j^{**} are now used to enter the force management table to obtain the number of operators at the board, namely σ_j^* and σ_j^{**}. The difference $(\Delta\sigma_j = \sigma_j^* - \sigma_j^{**})$ is the incremental number of board positions at time j.

Two further assumptions are now implicit:

(A3) The relative efficiency of the COSU is constant at its deter mined value over the whole day.

(A4) The actual board load, BL, will not be affected by a rate change.

Determination of incremental traffic expense. Given the incremental number of operators, the cost increment, Δ_k, for each day in each cardboard COSU will be estimated by

$$\Delta_k = \sum_{j=1}^{18} (c_j \cdot \Delta\sigma_j + C_j \cdot \Delta\tau_j)_k,$$

where

$c_j =$ the estimated cost per operator at the board at time j,
$\Delta\sigma_j =$ the change in the number of operators required at the board at time j,
$C_j =$ the estimated cost for personnel not at the board at time j, and
$\Delta\tau_j =$ the additional hours for personnel not at the board at time j.

A Traffic Office Cost Study was conducted to accumulate data used in estimating the cost of handling toll calls. A cost per employee per hour was developed, and within the model is applied to the incremental work units involved in handling increments in toll traffic, to derive operator costs directly attributable to toll increments.

The total cost of the additional operators at each board and of other necessary personnel is computed for an average business day (ABD), then accumulated over all switchboards within an operating area (OA). The total incremental cost in the operating area is then estimated for an average weekend day (WED). Summing the incremental costs for five ABDs and two WEDS then gives the total weekly OA incremental cost. The OA annual incremental cost is computed by summing the weekly costs over the 52 weeks centered on the study period. For this determination, forecasting techniques are also used. Adding the annual estimated OA costs over all 80 OAs gives the Bell System annual incremental cost of operating Traffic Offices.

The TSPS model. At the time of this research, there were too few TSPS COSUs in existence to support a separate TSPS model. Hence, the assumption is made that

(A5) The results obtained from a TSP model regarding operator cost structure and call processing abilities will apply to TSPS COSUs.

This assumption can be eliminated when sufficient experience and data are available to permit structuring of a TSPS model. In the meantime, the assumption appears reasonable because one would not expect the operator salary structure in TSPS offices to differ markedly from that in TSP offices.

For the most part, the TSP model is similar in structure to the cordboard model. However, there are enough differences to justify redefining the parameters and variables to be used:

$i =$ type of toll call (i = 1, 2,$\cdots$, 24),
$j =$ time period of the day (j = 1, 2,$\cdots$, 18),
$d_{ij} =$ demand for type i calls in period j,
$m_{ij} =$ incremental number of ij calls resulting from a demand change,
$s_i =$ number of seconds required to handle a type i call, including an adjustment for additives for assistance,
$L_j =$ total number of local exchange and intrastate seconds of operator work in period j,
$W_j =$ total number of interstate seconds of operator work in period j,
$V_j =$ total number of seconds of work in period $j(W_j + L_j)$, and
$\sigma_j =$ number of operators scheduled for period j.

Again the discrete function $\sigma_j(V_j)$ exists, but in this model V_j is in seconds rather than in work units.

As before, with W_j' as data, the computation

$$W_j'' = (1 + \pi_j)W_j'$$

is performed (using π_l as previously defined). Also,

$$V_j' = W_j' + L_j$$

$$V'' = W_j'' + L_j,$$

as for the cardboard model.

The cost increment ΔC for each day for each TSP COSU is computed as for the cardboard COSU. Weighting factors are again applied to account for days of the week and seasonality. The resulting annualized figures are summed over all TSP COSUs to obtain the total incremental traffic expense for TSP.

Directory Assistance expense model. The directory assistance (DA) model is a more aggregative model than either of the preceding. In addition, the assumptions upon which the model is based are not as intuitively appealing as those used in the preceding models. This model is based on the change in DDD traffic, because a change in DA volume is highly correlated with a change in DDD volume. The other models were based on operator-handled traffic change.

Since only toll DA is of interest, it is appropriate to make an assumption relating the change in DA volume to the change in toll calling volume. It is assumed that

(A6) The parameter a is the number of DA work units per toll call, and is not affected by a rate change.

The following variables are used in the mathematical representation of the model:

$D =$ total number of toll calls including DDD for an average business day, summed over all types of calls and time periods,

$M =$ incremental number of toll calls, including DDD, for an average business day,
$A =$ total number of DA work units for 1 day, and
$a =$ number of DA work units per toll call (A/D).

The values of D, M, and A are total values summed over all COSUs in the Bell System. An average expense per work unit in DA COSUs ($\bar{c}$) was developed from a pilot cost survey. For each day, the increment in cost to the Bell System because of a change in DA traffic is calculated as

$$\Delta W_{(DA)} = M_{\alpha}$$

$$\Delta E_{(DA)} = \bar{c} \cdot W_{(DA)}$$

where

$\Delta W_{(DA)} =$ incremental DA work units, and
$\Delta E_{(DA)} =$ incremental DA operator expense.

Weighting factors are applied for days of the week and seasonality, producing an annualized incremental traffic expense. The implicit assumption in this computation is that

(A7) No economies of scale will be realized in handling incremental DA traffic.

This justifies the linear form of the above equation. The assumption will be true, however, only when there are a large number of positions handling DA traffic.

Weighting to adjust for incremental equipment mix. The determination of the change in Traffic Office expense requires as input the number of operators on duty at all COSUs in the Bell System for all time periods of the day before and after a demand change. The traffic expense has been divided into three components for the model, representing the three major types of traffic equipment. Although there are relatively few TSPS positions in service at present, this is a more efficient technology. The future of 3CL versus TSPS — that is, the use of cordboards or the conversion of offices to more modern equipment — is an important factor in determining future expenses. To take account of future mixes, the model analyzes the demand increment first as handled by cordboard technology and then as handled by TSPS. On an aggregate basis, the model estimates how much of the incremental load will be carried by existing positions and how much by future positions. The results are then adjusted by growth factors to estimate expenses in the test year.

Other Traffic Expense (Accounts 621, 626, 627, 629, 630, 631, 633). To estimate Traffic Department expenses other than operator wages, statistical coefficients were derived relating incremental wages to incremental rest and lunch room expense, miscellaneous traffic expense, and house service (postage, printing, etc.) expense. The rationale for this procedure is that with an increase in operators and operator wages, requirements for employee facilities would vary in direct proportion, causing expenses to increase. For determining a change in the requirement for general traffic supervision, a factor is applied to the additional switchboards needed. The equations are:

$$\begin{aligned}
&\Delta\text{General Traffic Supervision \&} && = T_1 [\Delta \text{ Switchboards}] \\
&\quad \text{Operators EmploymentTrain.} && \\
&\Delta\text{Rest \& Lunch Rooms} && = T_2 [\Delta \text{ Operator Wages}] \\
&\Delta\text{Central Office House Svc.} && = T_3 [\Delta \text{ Operator Wages}]
\end{aligned}$$

where T_1 is the general traffic supervision expense factor, T_2 is the rest and lunch room expense factor, and T_3 is the central office house service factor.

Total incremental traffic expenses are now obtained by summing the Δ operator wages, Δ general traffic supervision including employment and training, Δ rest room and lunch room expense, and Δ central office house service.

Module 12 : Commercial Office expense. This module determines estimates of the expenses associated with the Business Office and marketing functions. Commercial expenses include the following:

(1) *Local Commercial Operation (Account 645).* Reported here are the salaries and office and traveling expenses of employees engaged in local commercial operations other than promotion or directory, such as preparing, changing, and handling miscellaneous customer transactions.

(2) *General Commercial Administration (Account 640).* Items reported in this category include the salaries and the office, traveling, and other expenses of employees in charge of the general administration of the Commercial Department.

(3) *Advertising (Account 642).* These are the expenses incurred in commercial advertising activities.

(4) *Sales Expense (Account 642).* Expenses incurred in canvassing for new business or for changing or reviewing existing service are reported in this account.

(5) *Connecting Company Relations (Account 644).* Expenses incurred in developing the interchange of business and other promotions of relations with counseling companies are included here.

(6) *Public Telephone Commissions (Account 648).* Amounts paid to the owners or tenants of premises upon which attended and non-attended public and semi-public telephone stations are located as general compensation for occupancy privileges, etc., are reported in this category.

(7) *Directory Expenses (Account 649).* Expenses incurred in preparing copy, printing, binding, and distributing directories are included here.

(8) *Other Commercial Expenses (Account 650).* All commercial expenses not properly chargeable to other accounts are totalled in this category.

Business Office Expense (Account 645). The major commercial expense that is associated with price changes is the expense associated with running the Business Offices. Here,

Bell System service representatives handle calls from subscribers related to their telephone bills — queries, complaints, and so on. These customer contacts are assumed to be sensitive in number to a change in billed interstate toll messages.

Data. The following are the variables required:

(1) *Endogenous variables*

The variables estimated by the model are the number of toll contacts, based on total accounts, total billed messages, and total service representatives; the amount of wages (total) paid to service representatives, based on total number of accounts, total number of toll contacts, and total service representatives; and the amount of commercial operations expense (total), based on total number of accounts, total number of toll contacts, and total service representatives.

(2) *Exogenous variables*

The module-exogenous variables are the incremental changes in billed messages as defined by the translation module.

(3) *Status variables*

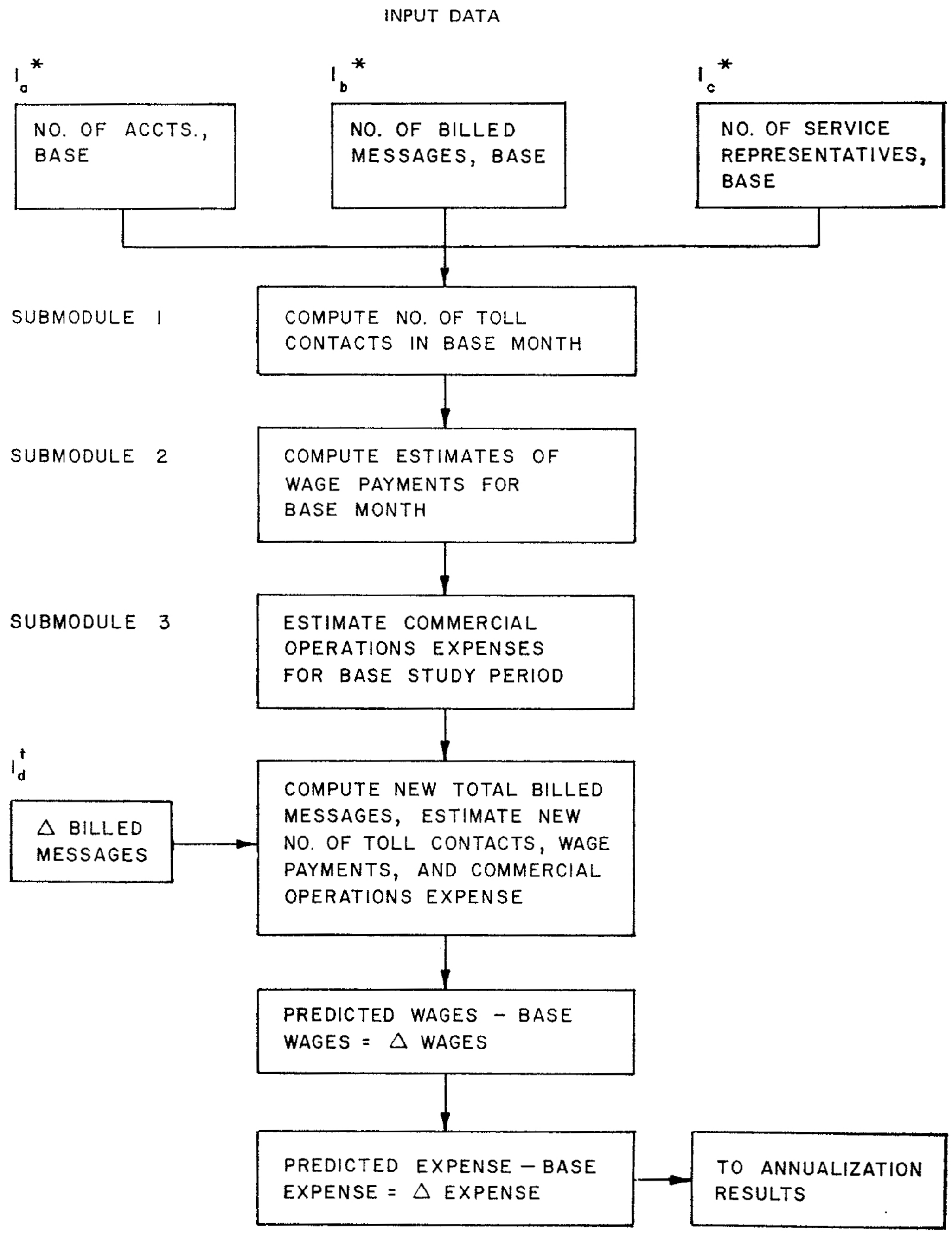
The status base comprises the base number of messages by major commercial office area throughout the Bell System; the number of customer accounts per area; and the number of service representatives per area.

Bell System Associated Company reports were used as data sources, together with expanded information which comprises monthly Business Office work measurement information, monthly analyses of commercial and marketing expenses, toll statistical data giving the interdepartmental billing results, and the like. The data were acquired as observations per month by operating area of each Company.

The model. Given a base status number of messages, plus the number of accounts and number of service representatives from the reporting forms cited above, and a Δ change in usage to adjust the status to a base study period, a sub-model estimates the number of toll contacts for the base study month. A second sub-model then estimates the wage payments made in the base study month, and a third sub-model estimates the commercial operations expenses for the base study period. Next, given a Δ change in billed messages (from the demand translation), the new total billed messages can be computed ($\Delta+$ base study messages) and used to estimate the number of toll contacts, the predicted wage payments for service representatives, and the predicted commercial operations expense for the revised load analysis. Figure 51 is a block diagram of the module.

FIGURE 51

COMMERCIAL EXPENSE MODEL, SIMPLIFIED BLOCK DIAGRAM



I_d^{tr} = INPUT FROM DEMAND TRANSLATION MODULE

* BASED ON PERIODIC ASSOCIATED COMPANY COMMERCIAL DEPARTMENT REPORTS

The model equations are:

(a) $T = a * A^{\alpha_1} * M^{\beta+\gamma_1} \log_e S$,
(b) $W = b * A^{\alpha_2} * T^{\gamma_2} \log_e S$,
(c) $C = c * A^{\alpha_3} * T^{\gamma_3} \log_e S$,

where
A = number of accounts,
T = number of toll contacts,
S = number of service representatives,
M = number of billed messages,
W = total wage payments,
C = total commercial operations expense, and
$\alpha_i, \gamma_i, \beta, a, b, c$ = estimated coefficients.

From (a) and (b), the incremental wages resulting from a Δ change in billed messages can be computed:

$$\Delta W = \text{Predicted } W - \text{Base } W.$$

The incremental commercial operations expense can be determined similarly:

$$\Delta C = \text{Predicted } C - \text{Base } C.$$

The models given here satisfactorily explain the relationships between the variables with small residuals and high multiple and partial correlation coefficients.

Other Commercial Department and related expenses. The model estimates the other incremental Commercial Department and related expenses resulting from a demand change by applying factors developed through statistical analysis of available time series data. The procedures are:

(1) *Marketing*

Statistical analysis was undertaken to develop the relationship be tween incremental revenues and incremental investment in Account 643 and the relevant portions of Account 645 over a ten-year period. The effects of local and state toll services were also examined for the same period. The following linear relationship based on these analyses is used in the model :

$$\Delta \text{ Market Expense } = F_1[\Delta \text{ Gross Revenues}],$$

where

F_1 is the marketing expense factor.

(2) *Advertising*

Analysis of accounting data for the same ten-year period was also used to develop the relationship between advertising and gross revenues. The factor resulting from this analysis is used in the following relationship to estimate advertising expense:

$$\Delta \text{ Advertising Expense} = F_2[\Delta \text{ Gross Revenues}],$$

where

F_2 is the advertising expense factor.

(3) *Independent Company Relations*

To estimate the incremental expense associated with Independent Company relations, a factor was developed based on a ten-year study of data pertaining to settlements and associated expense. The factor is applied to the change in Independent Company settlements produced in Account 644, as follows:

$$\Delta \text{ Independent Company Relations Expense} = F_3[\Delta \text{ Settlements}],$$

where

F_3 is the Independent Company relations expense factor.

(4) *Public Telephone Commissions*

To estimate the incremental expense associated with public telephone commissions, a ratio was developed by analyzing and relating Account 648 data for a ten-year period to toll coin revenues for the same period. The factor is applied as follows:

$$\Delta \text{ Public Telephone Commissions Expense } = F_4[\Delta \text{ Toll Coin Revenues}],$$

where

F_4 is the public telephone commissions expense factor.

(5) *Directory Expense*

No change associated with a change in toll usage is assumed in the directory expenses.

(6) *Other Commercial Expenses*

These (Account 650) are estimated as a function of the above incremental commercial expenses in the following manner:

Δ Other Commercial Expenses $= F_5[\sum \Delta$ Commercial Expenses (excluding Acct. 650)] where F_5 is the Other Commercial Expense factor.

Module 13 : General office salaries and expenses. This part of the analysis evaluates the following expenses which are sensitive to the demand for toll telephone service:

(1) *Accounting Department Expenses (Account 662).* This includes the salary, office, travel, and other expenses incurred by the Accounting Department. The estimates are developed in two procedures

   (a) Revenue Accounting, or the expenses associated with billing customers, which are directly affected by changes in toll demand, and

   (b) Other Accounting, or the expenses that are only indirectly affected (*i.e.*, work relating to payrolls, sundry disbursements, property and cost records, and corporate needs, developmental methods work, internal auditing, etc.).

(2) *Executive Department Expenses (Account 661).* This includes the salary, office, travel, and other expenses incurred by officers of the Company who are engaged in general administrative or management activities.

(3) *Treasury Department Expenses (Account 663).* This includes expenses associated with operations in the Treasury Department, such as debt and stock financing.

(4) *Law Department Expenses (Account 664).* This account represents the salaries and expenses of personnel in the Law Department and of legal counsel retained from outside the Bell System.

(5) *Other (Account 665).* This includes miscellaneous expenses of officers and other staff in general offices not properly chargeable to other general office accounts, where

$$
\begin{aligned}
\text{665-01} &= \text{Information Department,}\\
\text{665-02} &= \text{General Security, and}\\
\text{665-08} &= \text{Miscellaneous.}
\end{aligned}
$$

Revenue Accounting Expense (Account 662). The major expense within the category of general office salaries and expenses is associated with revenue accounting functions. These functions are carried out in approximately 108 Revenue Accounting Offices (RAOs) throughout the Bell System, and are defined as the preparation and sending of customer telephone bills. Each office is assigned a certain area in which it has billing responsibility. There is considerable variation in area size across the System.

Since some portion of the RAO function naturally pertains to toll message billings, the objective of this sub-module is to estimate the expenses incurred through RAO processing of billings specifically associated with toll usage increments. Because of the variation in office area size, as well as differences in organizational structure, degree of automation, and wage scale among offices, it is difficult to generalize the efficiency (and therefore the operating expenses) of the RAO for estimating purposes. Some of the offices are obviously technologically superior to others, not only at present but quite possibly for some time to come; in fact, it is not practical to try to forecast the speed with which automation will advance in those offices where present equipment may be hindering the efficiency with which billings are handled.

Considering the lack of detailed information and wide variation in structure, it seemed reasonable to develop for this sub-module a simple cross-section model of the revenue ac-

counting function; this approach precluded making forward-looking adjustments for RAO expenses, but otherwise proved sufficiently accurate for the first generation model.

Data. The data relevant to this accounting model are:

(1) *Endogenous variables*

These are the incremental revenue accounting expenses.

(2) *Exogenous variables*

Module-exogenous data in the form of incremental toll mes sages by major class of service are used. These pertain to DDD, operator-handled (station and person), and coin telephone calls.

(3) *Status variables*

The status of RAO offices is used, meaning the number of various types of messages which each is required to process under the "present" demand conditions (in the base year).

In the description which follows, the variables used will be:

$E =$ revenue accounting expense,
$M_A =$ AMA (DDD) toll messages,
$M_o =$ Operator-handled toll messages,
$M_c =$ Coin toll messages,
$B =$ Number of bills rendered,
$S =$ Number of service orders received, and
$Q_t =$ Time (in base year quarters) with first quarter 1967 = 1, second quarter 1967 = 2, and so on.

The model. The functions of an RAO involve charging the customer for both the basic monthly service and local message-unit type calls and for toll calls made during the billing period. The question that this model is designed to answer is: What would be the estimated long-range revenue accounting expenses associated with billing customers for the incremental toll messages which they might place as a result of a given rate change? This question must be considered in the context of other functions performed in RAOs, such as service order activity.

The following models were investigated, using available data on an RAO basis:[24]

$$E = a_o + a_1 M_A + a_2 M_o + a_3 M_c, \tag{1}$$

$$E = a_o {M_A}^{a_1} \cdot {M_o}^{a_2} \cdot {M_c}^{a_3}, \tag{2}$$

$$E = a_o {M_A}^{a_1} \cdot {M_o}^{a_2} \cdot {M_c}^{a_3} \cdot B^{a_4} \cdot S^{a_5}. \tag{3}$$

[24] The basic data source was the RAO Comptroller's Department Production and Cost Results quarterly reports for 1968, which provided information on toll and general accounting operations.

Model (3) ran into serious estimation difficulties. B was highly correlated with M_A, M_o, and M_c, and so was S.

We then considered including the dummy variable Q_t, representing the points in time at which the repeated cross-section sets of observations are taken. For instance, if observations are taken for 100 RAOs in six different quarters, the sample will have 600 observations. The purpose of introducing an explicit term for time is to account for any possible trend in the dependent variable, and thus to explain possible bias due to the time lapse between observations. This model is:

$$E = a_0 {M_A}^{a_1} \cdot {M_o}^{a_2} \cdot {M_c}^{a_3} \cdot B^{a_4} \cdot S^{a_5} \cdot {Q_t}^{a_6}.$$

All four of the models were then investigated on an Operating Company basis, with RAOs aggregated by company. The second model was selected for inclusion in the incremental cost model.

Let $E(I)$ and $E(II)$ denote estimates of total expenses associated with toll accounting functions under the two rate structures. Then,

$$\log_e E(I) = (a_0 + a_1 \log_e M_A + a_2 \log_e M_O)_I,$$

and $E(I)$ is obtained by taking the antilog of the left-hand side. Similarly, $E(II)$ is calculated by taking the antilog of the left-hand side of the equation for the alternative rate structure. Finally, the total additional expense is

$$\Delta E = \big[E(I) - E(II)\big],$$

and ΔE is annualized by applying an appropriate seasonal factor,[25] on the assumption that the price and demand changes would not affect seasonal relationships. (It is recognized, however, that in actuality rate changes could alter seasonal relationships.)

Estimates of incremental general office salaries and expenses. Estimates of incremental general office salaries and expenses for Accounts 661 through 665-08, excluding the Revenue Accounting portion of Account 662, are derived by applying ratios to incremental wages for Maintenance, Commercial, Revenue Accounting, and Traffic Office functions. The equation is of the general form

$$\Delta \text{ General Expenses on Wages } = A * \Delta W$$

where

A = average expense ratio.

Individual ratios were developed by studying the relationship of incremental expenses in the four areas to incremental wages in the same areas over a base period. Since the ratios appeared to be relatively constant over the four office categories, an average is presently

[25] This factor was developed from quarterly data pertaining to the base period.

being used to estimate these expenses. Expressed in Bell System accounting categories, the equation becomes

$$\begin{aligned}\Delta\text{Expenses} = \Delta\Big[&661 + 662(\text{other})\\ &+ 663 + [664; 665.01; 665.02; 665.08]\\ &+ \text{associated relief and pensions}\\ &+ \text{associated Social Security taxes}\Big],\end{aligned}$$

or in symbolic notation,

$$\Delta W = \Delta M_W + \Delta C_W + \Delta R_W + \Delta T_W,$$

where

$$\begin{aligned}\Delta W &= \text{incremental wage expense,}\\ \Delta M_W &= \text{incremental maintenance wages,}\\ \Delta C_W &= \text{incremental commercial wages,}\\ \Delta R_W &= \text{incremental other RAO wages, and}\\ \Delta T_W &= \text{incremental other traffic wages.}\end{aligned}$$

For Maintenance wages,

$$\Delta M_W = (602) * F_1 + (604.01, 57C) * F_2 + (606) * F_3 + (603) * F_4.$$

For Commercial wages,

$$\Delta C_W = (640 + 642 + 644 + 648) * F_5 + 645.$$

Revenue Accounting wages are part of the output from the Revenue Accounting Model described in part A of this module. For Traffic wages,

$$\Delta T_W = (621 + 626 + 630) * F_6 + 624 + 627.$$

$F_i (i = 1, 2, \cdots 6)$ are factors derived externally by analysis of available data, and are exogenous inputs to the computation of wage or labor expenses.

Module 14: Maintenance expense. Essential to the study of long-run incremental costs are the expenses associated with repairs, rearrangements, and changes of plant in service, which are recorded separately for the various types of plant and of central office equipment and station equipment, and for buildings and grounds. Test desk expenses, incurred in the testing of circuit equipment, are also included under the general category of maintenance costs. Each of these expense categories will be described briefly as it is analyzed in the model.

Switching maintenance. Incremental maintenance expense 1s computed on a switching frame basis for the various incremental pieces of switching equipment — step-by-step,

intertoll selector and trunk frames, and No. 3 type manual toll switchboards and TSPSs estimated in the earlier capital investment modules. The incremental expense associated with each type of equipment was developed through analysis of quarterly maintenance data obtained from the Associated Companies.

Data. The data are the following:

(1) *Endogenous variables.*

The model develops estimates of incremental maintenance work units, by type of incremental switching equipment and by upkeep and change categories; and incremental annual switching maintenance expense.

(2) *Exogenous variables.*

Exogenous to this module are the number of incremental frames by type of switching equipment (derived in the switching module). External to the overall analysis are the number of upkeep and change work units per incremental frame by type of switching frame, and the expense per incremental work unit by type of switching frame.

Procedure. Models were formulated for each type of switching frame based on the number of upkeep work units per frame and the expense per upkeep work unit, to estimate maintenance hours per frame and related maintenance expense. Table 15 gives these values for some types of frame associated with No. 4 Crossbar equipment.

TABLE 15. ANNUAL MAINTENANCE EXPENSE PER FRAME
(TYPE OF EQUIPMENT: NO. 4 CROSSBAR)

TYPE OF FRAME	NO. OF UPKEEP WORK UNITS	DIRECT LABOR	OTHER*	TOTAL
INCOMING LINK				
ALL EXTENSIONS	129.6	219	318	537
OUTGOING LINK				
ALL EXTENSIONS	129.6	219	318	537
JUNCTOR GROUPING	X			
SENDER LINK	185.6	212	180	392
LINK CONTROLLER				
AND CONNECTOR	60.8	69	59	128
INCOMING SENDER	197.6	225	192	417
OUTGOING SENDER	197.6	225	192	417
CAMA SENDER	278.4	317	270	587
MARKER CONNECTOR	60.8	69	59	128
DECODER CONNECTOR				
SUPPLEMENTARY	60.8	69	59	128
TRANSLATOR EQUIPMENT				
HOME	480.0	811	1176	1987
FOREIGN AREA	480.0	811	1176	1987
EMERGENCY	480.0	547	466	1013
TRANSLATOR CONNECTOR				
FOREIGN AREA	60.8	69	59	128
SUPPLEMENTARY	60.8	69	59	128
EMERGENCY	60.8	69	59	128
SUPPLEMENTARY	60.8	69	59	128
BLOCK RELAY	51.2	58	50	108
TRANSVERTER – CAMA	120.0	137	116	253
TRANSVERTER CONNECTOR–				
CAMA	60.8	69	59	128
TRUNK CLASS TRANSLATOR–				
CAMA	180.8	206	175	381
SUPPLEMENTARY	129.6	148	126	274
RECORDER – CAMA	120.0	137	116	253

* SEE FOOTNOTE ON TABLE 13.

x NO WORK UNIT VALUE.

Status data used in these models were acquired from a selected number of the Associated Companies and Long Lines. From these data, estimates of the expenses associated with maintaining local equipment and toll switching equipment were drawn. Also, a Bell System average maintenance expense for local and toll equipment combined was derived. The average annual maintenance expense comprises an expense per frame and an expense per upkeep work unit, for direct labor and for the combined other charges.

The numbers of additional frames of equipment by major switching category are estimated in the dial and manual switching modules, described earlier. Incremental switching maintenance expenses are estimated by multiplying these frames by the average number of upkeep work units per frame by switching class and by the expense per upkeep work unit as follows:

$$SwM_{1,k} = F_k E_{1,k} (U/F)_k$$
$$SwM_{2,k} = F_k E_{2,k}$$

where

$SwM_{1,k} =$ Switching maintenance labor expense by equipment class k,
$SwM_{2,k} =$ Switching maintenance non-labor expense by equipment class k,
$F_k =$ incremental frames of type k,
$k =$ Step-by-Step, Crossbar, Manual, and ESS,
$E_{1,k} =$ labor expense per upkeep work unit for type k,
$E_{2,k} =$ non-labor expense for type k, and
$(U/F)_k =$ upkeep work units for type k.

Other Maintenance Expenses. These expenses are associated with Account 602 (repairs to outside plant); Account 603 (repairs to central office circuit and radio equipment), and Account 606 (repairs to building and land). Part of the incremental maintenance expenses associated with demand changes include the costs of repairing, maintaining, rearranging and changing equipment.

Data. The following data pertain to this portion of the module:

(1) *Endogenous variables*

Estimates of the expense involved in maintaining outside plant (Accounts 241-244); central office equipment (circuits and radio, Account 221-57C); and buildings (Account 212) are developed. Within outside plant, incremental maintenance expenses were estimated and expense ratios developed for aggregate classifications of facility: coaxial cable, N and T carrier, and radio. The outside plant and central office circuit equipment maintenance expenses are associated with repairs and upkeep, plus maintenance of circuits and radio equipment and re arrangement of cable and supporting structures. Within the land and buildings account, the expenses of interest are those associated generally with the care of grounds, hallways, and other public areas.

(2) *Exogenous variables*

The incremental investment in circuits by facility type is developed in earlier modules. These investments are now used as module-exogenous data to develop estimates of circuit maintenance expense. Expense factors derived through a time series analysis of Operating Company data are also used as exogenous variables, applied to the appropriate incremental capital costs.

The model. The equation is

$$M_{\gamma l} = \sum_{i=1}^{3} g_{i\gamma l}\left[\Delta I_{i\gamma}\right],$$

where

$M_{\gamma l}$ = incremental maintenance expense associated with incremental capital investment account (γ) and expense class (l),

γ = outside plant maintenance, central office equipment circuit maintenance, central office equipment radio equipment,

l = labor, building maintenance,

$\Delta I_{i\gamma}$ = incremental investment of facility i (from the capital investment modules),

$g_{i\gamma l}$ = ratio of incremental expense to incremental capital investment for facility i, where

i = N & T carrier facilities, L coaxial facilities, or radio facilities.

Circuit testing expense. Circuit testing expenses include costs incurred in receiving and recording reports of trouble on circuits; the costs of testing, monitoring, and adjusting toll circuits to maintain such circuits in proper order; and costs incurred in the determination, location, and remedying of trouble.

Data. These are the variables associated with this sub-model:

(1) *Endogenous variables*

Incremental circuit testing expenses are estimated in this procedure.

(2) *Exogenous variables*

From the capital investment modules described earlier, the incremental circuit miles (by 11 circuit mileage bands by facility type), the incremental circuits (by 11 circuit mileage bands by facility type), and coefficients relating circuit testing expense to capital investment (cost per mile and cost per circuit by facility type) are used. The coefficients include wage loading-relief and pensions, Social Security payments, and general expenses.

The model. Coefficients for testing expense are estimated on the basis of expense dollars per circuit and expense dollars per mile. They are applied to the incremental circuits and mileage by circuit facility type and by length of circuit group. The procedure is to determine the differences between estimated annual testing expenses with and without a demand change. The differential represents the incremental expense, including not only

changes in wages and minor overhead items for staff at test desks, but also the changes in the number of miles and in the number of circuits tested. The model is

$$CT = \sum_i \alpha_i m_i + \sum_j B_j n_j,$$

where

$$\begin{aligned} CT &= \text{circuit testing expense,} \\ a_i &= \text{cost per mile for the ith facility,} \\ m_i &= \text{mileage in ith facility group,} \\ B_j &= \text{cost per circuit injth mileage band, and} \\ n_j &= c\text{ircuits injth mileage band.} \end{aligned}$$

Module 15: Other operating expenses. Other operating expenses are associated with a number of miscellaneous categories:

(1) *Insurance (Account 668).* In this category are the expenses associated with the purchase of commercial insurance which protects the company against loss and damage in its telephone operations.

(2) *Accidents and Damages (Account 669).* Items in this account relate to occurrences such as death or injury or damage to property of others which are related to the company's telephone operations.

(3) *Operating Rents (Account 671).* This includes rents payable for use of space in buildings occupied by the company, space in conduits, pole line space, equipment, and so on.

(4) *Relief and Pensions (Account 672).* Pensions and other benefits paid to active and retired employees, as well as salaries and expenses incurred by the Company in conducting its relief and benefit programs and in operating its general medical departments are included in this account.

(5) *Telephone Franchise (Accounts 673 and 676).* These accounts are for the recording of costs associated with furnishing services, plant, material and supplies to municipalities or other government authorities, in compliance with franchises, ordinances, or similar requirements, for which no reimbursement or payment is received by the company.

(6) *Other Expenses (Account 675).* This includes all operating expenses not properly charged to other accounts.

(7) *Engineering (Account 665.09).* This includes the costs of making engineering studies of a general nature, not applicable to any particular project and especially not related to construction.

Procedure. To estimate the incremental general expenses associated with a toll demand change, the incremental relationships between Account 100.1 (plant in service) and Accounts 665.09, 668, 669, 671, 672, 673, 675, and 676 were studied for a 12-month base period. The following relationships are used to estimate these expenses:

$$\Delta E_0 = A_2 \Delta I_{100.1},$$

and

$$\Delta E_{RP} = A_3 \Delta W_i,$$

where

$$\begin{aligned}
\Delta E_0 &= \text{incremental operating expense,}\\
\Delta E_{RP} &= \text{incremental relief and pensions expense,}\\
A_2 &= \text{factor relating } \Delta \text{ other operating expenses to } \Delta \text{ investment}\\
&\quad\ \text{Account 100.1,}\\
A_3 &= \text{actor relating } \Delta \text{ relief and pensions } \Delta \text{ wages,}\\
\Delta W_i &= \Delta \text{ wages as defined in Expense Modules 11 through 14,}\\
&\quad\ \text{and expressed in terms of the Bell System Standard Accounts designations,}\\
E_o &= \text{Account } 665.09 + 668 + 669 + 671 + 673 + 675 + 676,\\
&\quad\ \text{and}\\
E_{RP} &= \text{Account 672.}
\end{aligned}$$

The change in plant investment (Account 100.1) on which these computations are based is calculated as

$$\Delta I_{100.1} = \sum \text{Incremental Capital Investment [Land; Buildings; Circuit Equipment; Switching Equipment; Exchange Plant; Furniture and Office Equipment; Motor Vehicle Investment].}$$

Module 16: Annualization of costs and revenues. In the final section of the analysis, estimates of average annualized incremental costs are developed and compared with annual incremental revenues. Through this comparison, one can determine how the changes in revenue requirements compare with changes in revenue for the study period. That is, if the change in annual revenue requirements for some period is

$$\begin{aligned}
\Delta RR = &\text{(Amortization Factor)*}(\Delta \text{ Capital Investment})\\
&+\Delta\text{Annual Operating Expense}\\
&\qquad + \Delta\text{Planned Rate of Return*(Rate Base),}
\end{aligned}$$

and if the change in annual revenue for the System at the same period is ΔR, then the present worth values $PW(\Delta RR)$ and $PW(\Delta R)$ may be compared.

Annual costs are developed on two bases:

(a) On an economic basis, assigning costs to jurisdictional entities on the basis of ownership of facilities and by type of demand (inter- vs. intrastate), and

(b) On a separations basis, employing the procedures[26] presently in use for rate cases and division of revenues.

[26] These are procedures that have been developed for allocating costs between intrastate and interstate jurisdictions, and are acceptable to both state and Federal regulatory bodies. Their function is to determine what part of the total Bell System revenue requirements is to be obtained from interstate rates and what part from intrastate rates.

For rate filings, revenue and cost data are generally furnished for a "test period." This period can in theory be defined as the planning period. However, for rate planning purposes the revenues and costs for the planning period are usually equated to average annual estimates, which are then identified as the "test period" data.

The incremental capital investment and incremental operating expenses derived in the preceding parts of the analysis are brought together and combined with other costs and expenses that are not previously estimated but still integral parts of an average long-run incremental cost computation. These costs are then used to develop average estimates of incremental costs for a test year, and are dis played in the form of investment and expense reports. Various levels of output are then derived; two of these are based on the Bell System Standard Accounts as defined in the uniform system of accounts for Class A Telephone Companies.[27] This part of the analysis consists of basic submodules, to provide flexibility both in design and in future model refinement.

Submodule 1: Revenues. The major areas of incremental revenues attributable to toll switched network services are summarized for an average test year. (The demand forecast modules estimate average annual increments for the planning period only.

The following accounts are summarized in this section:

Account 510 —	Message Telecommunications Service
	Interstate
	Intrastate
Account 511 —	Wide Area Telephone Service
	Full Time
	Measured Time

Submodule 2: Capital investment annual costs. The capital in vestment estimates previously computed are mapped to their respective accounts. In this submodule, annual costs are developed in the form of annuities. The series derived for various alternative pricing structures have been in the context of a uniform planning horizon. Present worth methods have been employed to satisfy the basic requirement that alternatives can be compared (*i.e.*, money consequences are measured at a common point in time, and the net discounted value of the prospective cash flows is used as a measure of the relative economic attractiveness of the proposed investment).

To compute annual costs, the investment series is converted to an annuity (*i.e.*, equivalent uniform series) by use of appropriate com pound interest factors. The investment series were formed from irregular cash flows, and the individual investments are first discounted and summed. That is,

$$\Delta V = \sum_{t=0}^{N} \Delta I_t (1 + \tau_t)^{-t},$$

[27] These are companies with annual operating revenues exceeding $250,000.

where

$$\Delta V = \text{present value of a sum of investment,}$$
$$\Delta I_t = \text{investment for period } t,$$
$$\tau_t = \text{effective interest rate for period } t \text{ (tax-adjusted rate of return), and}$$
$$N = \text{amortization period.}$$

To annualize this cost, it is assumed that estimates of annual costs are equivalent uniform annual costs. The cash flows are then converted to an equivalent uniform series by use of appropriate compound interest factors. To find the equivalent value of the uniform cash flow series, an annuity is computed; this is an amount equivalent to the present sum of money that will yield for one year a stated rate of return per compounding period. That is, the following form (due to Sable[28]) will be assumed:

$$\Delta V' = \Delta V \left[\frac{\gamma(1+\gamma)^N}{(1+\gamma)^N - 1} \cdot \frac{1}{1-\tau} - \frac{\tau/N}{1-\tau} \right],$$

where

$\Delta V'$ = annual or uniform cash flow taking place at the end of each and every period for N periods,

ΔV = as defined above,

τ = tax rate,

γ = tax-adjusted rate of return, and

N = amortization period.

[28] Sable, E. G., "Engineering Economics – A New Method of Computing Revenue Requirements"; Bell Telephone Laboratories technical memorandum.[7] The formula described by Sable has been adopted because it seems a reasonable one, although there has been considerable controversy as to what constitutes an appropriate formula for computing revenue requirements.

The following estimates are derived in the above manner

Account	Title	Source
221–17C	Central office — Manual Switching Equipment	Manual Switching Equip. Module
221–37C	Central office — Dial Switching Equipment Step $\times$ Step	Dial Switching Equip. Module Other Switching Equip. Module
221–47C	Central office — Dial Switching Equipment Crossbar	Dial Switching Equip. Module Other Switching Equip. Module
221–77C	Central office — Dial Switching Equipment Electronic	Dial Switching Equip. Module Other Switching Equip. Module
221–57C	Central office — Circuit Equipment	IX Circuit Module Toll Connect Circuit Module
221–67C	Central office — Radio Circuit Equipment	IX Circuit Module Toll Connect Circuit Module
241–244	Outside Plant — Circuit Equipment	IX Circuit Module Toll Connect Circuit Module
261	Furniture and Office Equip.	Other Capital Investment Module
264	Vehicles and Other Work Equipment	Other Capital Investment Module
211	Land	Σ [Land & Buildings computed in Switching Modules + Circuit Modules]
212	Buildings	

Submodule 3: Annual operating costs (expenses). The incremental expenses computed in the labor and expense modules are summarized here to provide estimates for a test year. The operating expenses comprise many categories, including the costs involved in maintaining plant, marketing the services offered, and administering the overall policies and activities of the business. They are computed based on the assumption that the demand is an additive increment for each year in the planning period. Therefore, they are constant other than adjustments for inflation in the usage rate portions.

Annual expenses are computed by converting the expense flows to an equivalent uniform series by use of appropriate compound interest factors similar to the procedure described in the module for estimates of annual capital investments. The following accounts are handled in this manner:

602	Maintenance — Repair pole lines, cable, etc.	Maintenance Exp. Module
603	Test Desk Work	
604	Central Office Equip.	
606	Building & Ground	
621	Traffic — General Supervision	Traffic Expense Module
624	Operator Wages	
626	Rest & Lunchroom	
630	Central Office House Service	
640	Commercial — General Administration	Commercial Module
642	Advertising	
643	Sales	
644	Connecting Company Relations	
645	Local Commercial	
648	Public Telephone Commissions	
661	General Office Expenses — Executive Dept.	Estimated as one value
662	General Accounting	
663	Treasury	
664	Law	
665.01,02	Information & General Secretary	
662	Revenue Accounting	Revenue Accounting Module
668	Other Operating Expenses — Insurance	
669	Accidents and Damages	
671	Operating Rents	
665–09	Engineering	
672	Relief and Pensions	

Submodule 4: Separations analysis. Telephone companies furnish both intrastate and interstate toll communications services, the former falling under the jurisdiction of the state regulatory commissions and the latter under the Federal Communications Commission, This division of regulatory responsibility was discussed in an earlier section.

A large portion of telephone company property is used in common for both intrastate and interstate services. In accounting for telephone plant and the expenses of operating the business, it is not feasible to determine the actual costs applicable to intrastate services and those applicable to interstate services. Separations procedures acceptable to all of the state and Federal regulatory bodies have therefore been developed for allocating costs between the relevant jurisdictions. These procedures are needed when a filing for a change in rates is made before one of the commissions, to support testimony of the Company. They are also used in the division of revenues among the Companies and Long Lines.

The Separations Manual outlines methods for this cost allocation which are deemed to be fair and equitable. The procedures reflect the view that the Associated Companies, Independent Companies, and Long Lines are partners engaged in jointly furnishing communications services. Under this arrangement, each of the partners does a certain amount of operating work and furnishes some of the various types of telephone plant. The procedures also attempt to account for variations in the costs and relative proportions of plant investment and operating effort involved in furnishing services.

The rates charged to customers are specified in detail in tariffs filed with the regulatory commissions, intrastate tariffs with the state bodies and interstate with the FCC. The method of making price changes in these tariffs varies among jurisdictions, depending on the applicable laws and rules of procedure. Rate cases are almost always decided on the basis of a commission's establishing for a "test period" a "rate base" in dollars and an allowable "rate of return" on that base. It is therefore necessary in studying price-cost relationships to be able to estimate the effects on the regulatory jurisdictions, and that is why the separations procedures were modeled.

1. *Local exchange plant separations.* The effects of the separations procedures on the exchange plant costs and associated expenses incurred by the Bell System as a result of a demand change are estimated by a method which is basically an accounting procedure following the rules set forth in the Federal Communications Commission Report and Order of January 29, 1969, on Docket No. 17975, and in the Separations Manual. It is assumed that the demand change which would result from the setting of new toll rates would require no re engineering of the local exchange plant (other than as described in preceding sections), so that in this area the incremental costs and expenses assigned to interstate and intrastate usage would be created mainly through the application of the separations procedures. This assumption is fairly realistic, since it has been observed that the local exchange plant facilities are, for the most part, insensitive to toll traffic. Further, the approach generally makes the model simpler both in conception and in implementation.

Data. The following data are required:

(1) *Endogenous variables.* Investment separations are estimated for 12 categories of plant, by Division of Revenues area[29] —

Exchange trunk circuit equipment
Exchange outside plant
TTY station apparatus
TTY station connections
Telephone station apparatus
Telephone station connections
Radio station apparatus
Radio station connections
Large private branch exchanges
Message exchange trunk outside plant (KCT-2)
Message exchange trunk circuit equipment (KCT-2)
Local dial switching equipment

Expense separations are determined for the following expense categories —

Central Office Equipment Depreciation
Outside Plant Depreciation
Stational Equipment Depreciation
Total Exchange Plant Depreciation

[29] Division of Revenue Areas, of which there are 57, are geographic designations of Operating Company territory used for administering the separations procedures.

Central Office Equipment Maintenance
Outside Plant Maintenance
Station Equipment Maintenance Total
Exchange Plant Maintenance
Operating Rents
Taxes on Book Costs and Telephone Franchise Requirements
Commercial Department Expenses
General Expenses based on "Big 4" Expenses
General Expenses based on Total Exchange Plant
Miscellaneous Income Charges
Relief and Pensions
Social Security Taxes
Summaries by DR Study and Toll Center Areas

(2) *Status variables.* The model requires base toll usage data for determining the change in demand. These inputs are —

Base interstate minutes of use (MOU) by toll center area (TCA) from the March Point-to-Point Study data.

Base minutes of use of intrastate toll, interstate toll, and exchange (local) by Division of Revenue (DR) study area as reported to AT&T on forms submitted monthly by the Associated Companies. Also from the same sources, KCT-2 category long distance and total MOU; dial equipment for inter- and intrastate toll and local MOU; and the composite station rate and KCT-2 plant ratios (which will be described).

In addition to the assumption that the exchange plant will not be re-engineered as a result of a demand increment, it is also assumed that the composite station rate ratios will not change. This factor is required in the computation of separations for DR areas. It represents the DR area's average station rate for a call having an average length of haul (LOH) divided by the nationwide average station rate for a call with an aver age LOH. Another assumption is that the intrastate toll and exchange MOU will not change. Only interstate usage will be assumed to change in reaction to a new toll rate plan. Further, it is assumed that the average of the monthly separation factors for a given DR area over a year equals the one month's factor that is actually computed.

The analysis requires base book costs for state, long distance, and local for the 12 categories of plant (by Division of Revenue areas) listed under endogenous data. Each of the 57 DR study areas reports monthly to AT&T certain data such as subscriber line usage in various categories, lengths of haul of messages, and so on. These provide a basis on which the aggregated toll center area (TCA) MOU (also filed by DR area) can be adjusted.

(3) *Exogenous variables.* Module-exogenous data required are the incremental interstate MOU by TCA from the demand forecast models, via the translation module. External data include inflation factors, used to simulate the growth in each DR area that would occur from the date of the book cost data to any specified time; depreciation ratios

for central office equipment, subscriber plant, and station equipment; and maintenance ratios for central office equipment, outside plant and station equipment.

Procedure. To correlate TCA data with DR area designations, the TCA base period and incremental MOU are aggregated by their respective DR areas, each TCA usage data record having earlier been keyed to a DR number. These totals are processed by DR areas, with plant book costs computed for each; the costs in each plant category or total are then prorated in proportion to the base MOU reported for each TCA. By this method, the model determines the closest possible approximation of incremental exchange plant costs per TCA.

The ratio of the TCA minutes in each DR area to the base inter state minutes of use is applied as a factor to each of ten categories of minutes, yielding an approximation of what the minutes in each category would be for the TCA data base adjusted to the DR level.

The separations factors for exchange plant by Bell System and by DR study area are computed based on the inflation and correction values used in actual Division of Revenues separations each month. (These are corrections for load-dependent equipment usage which is not otherwise accounted for in the MOU data, such as incomplete dialing. The exchange plant book costs are .first adjusted for growth. Then the adjusted costs are multiplied by the separations factors to determine the costs assigned to interstate both before and after the demand change, for the 12 categories of exchange plant. The amount associated with the incremental demand is given separately.

These assigned costs are totaled by DR area and summarized for the Bell System. Each DR area's total cost is then prorated to the TCAs in that DR area using the proportion of TCA base minutes to the total DR base minutes.

The model uses this same procedure to compute the expenses associated with the specified interstate toll usage. Expense factors are developed for each major plant category and DR study area, by application of ratios which are the percentages of the total exchange plant book costs falling into each category.

Expenses in each category are adjusted first for the given amount of interstate toll usage, based on the adjusted book costs for that usage. Then the expenses assigned to interstate for the base amount of traffic are determined, followed by the new expenses assigned to inter state. By subtraction, the difference (which represents the change in separations due to the incremental usage) is computed.

Depreciation ratios for central office equipment, subscriber plant, and station equipment, and maintenance ratios are then applied to the corresponding plant accounts. These ratios were all developed based on telephone company experience. Totals by DR area and by TCA are then carried over to the final revenue-cost analysis.

2. *Interchange switching equipment separations.* Interexchange switching machine investment is computed by approximating separations procedures as in the previous section.

Data. The following are the data requirements:

(1) *Endogenous variables.* The separations base, adjusted for the incremental investment resulting from a change in demand (by DR area), is produced.

(2) *Status variables.* Base separations data are required by DR area, structured for three categories of switching equipment —

Manual,
Dial, and
AMA.

These data comprise the total investment in each category and the amount in each category assigned to interstate. Base inter state and intrastate minutes of use by DR area by equipment class are also used.

(3) *Exogenous variables.* Module-exogenous information required is the incremental interstate and intrastate minutes of use by DR area.

Procedure. The status data are used to develop a theoretical total usage value which was not otherwise readily available. The base plant investment for each category of equipment is divided by the amount of that investment that is assigned to interstate, to obtain an inflation factor. This factor is applied to the interstate usage data that represent the status within each DR area, by equipment class; these data when "inflated" then become the theoretical total MOU in each category.

The incremental change in MOU is then added to this theoretical total, and the change in percent usage assigned to interstate computed. The new percent assigned to interstate when applied to the new base gives a new interexchange switching investment assigned to interstate (and, by inference, to intrastate). The incremental investment is derived by subtracting the old base from the new.

The change in interstate investment is also computed. Investment is split between Associated Company owned equipment and Long Lines owned; since Long Lines equipment is assigned 100 percent to interstate, no factor is needed. To the Δ interexchange switching equipment estimated for the Associated Companies, the new percent interstate investment is applied; this results in a new estimated Associated Company investment in equipment assigned to interstate.

Summing (1) the base investment in interstate by DR area, (2) the Long Lines incremental investment, and (3) the Associated Company incremental investment yields the incremental switching equipment investment assigned to interstate.

3. *Interexchange circuit facilities separations.* If the interexchange (IX) circuit requirements for carrying interstate toll traffic are affected by a demand change, the separations of costs associated with these facilities are also changed. Within this module, as with the other separations, the steps which lead to the computation of the new book costs assigned to interstate usage are based on accepted Bell System separations procedures.

Separations are computed for each DR area by traffic category, of which there are eight:

(1) Long Distance interstate (LDI) MTS usage, both Long Lines layout (LLL)[30] and Associated Company Layout (ACL),

(2) Jointly used (state-interstate) MTS usage,

(3) Long Distance state (LDS) MTS usage,

(4) TWX RX loops,

(5) TWX SP trunks,

(6) Private Line LDI and LDS,

(7) Long Lines order and alarm circuits, and

(8) Circuits rented to others.

Data. The following information is involved in this separations procedure:

(1) *Endogenous variables.* The results of the procedure are given as the change in separations for interexchange circuit investments in the following categories of plant:
Outside Plant (Accts. 241-244)
Circuit Equipment (Acct. 221-57C, other than radio)
Circuit Equipment (Acct. 221-67C, radio)

(2) *Status variables.* Book capital investments by DR areas by traffic classification, TX circuit miles by DR area, and message minute miles (MMM) on jointly used circuit groups[31] are used to represent the status.

(3) *Exogenous variables.* Module-exogenous variables include the incremental MMM values for:
LD circuit groups (LLL),
Associated Company circuit groups (ACL), and
Jointly used circuit groups (LDS).

Procedure. The base period book cost and IX circuit mile data are used to calculate an embedded status book cost per circuit mile. The changes (Δ) in investment and circuit mileage produced by the model for the LDI and joint (state/interstate) categories of traffic are added to the base, and the investment per circuit mile on the revised total values are calculated.

Using the base MMMs pertaining to interstate and state usage for appropriate traffic categories of service [such as category (2)], the MMMs by the total. The changes (Δ) in interstate MMMs produced by the toll connect and intertoll circuit modules are then used to adjust the interstate usage percent. [Note: The circuit modules estimate changes in toll

[30] LLL does not necessarily mean Long Lines ownership, but for the model it was assumed to have this meaning.
[31] These groups are further classified as interstate only or intrastate and joint for each DR area.

connecting and TX trunk facilities requirements by toll center areas. The changes in Associated Company plant used for both inter- and intrastate traffic must be apportioned over DR areas for the separations analysis. Since actual mileage for each trunk group associated with a DR area is not available, a statistical procedure was developed to approximate this allocation.]

For the circuits that are wholly interstate [category (1)], the base (embedded) book costs are subtracted from the new book costs, and the differences are the Δ changes in book costs assigned 100 percent to LDI.

The base MMM interstate percentage is applied to the base investment for jointly used circuits, which produces a base cost assigned to LDI. The adjusted MMM interstate percentage is then applied to the revised jointly used circuit investment to produce estimates of the revised investment assigned to LDI. By subtracting the base investment assigned to LDI from the revised investment assigned to LDI, the Δ separations investment of category (2) assigned to LDI is obtained.

The circuit miles assigned to LDI for categories (4) through (7) are multiplied by the new cost per circuit mile. The addition of the three results [for categories (1), (2), and (4) through (7)] produces the net effect on interstate investment for the Associated Companies. The change in LLL circuit investment is assumed to be 100 percent assigned to interstate. When the Long Lines effect is added to the Associated Companies effect, the total Bell System change in TX circuit investment assigned to interstate is determined.

The output comprises the change in separations for interexchange circuit investment that results from a structured incremental demand. In this portion of the model, outside plant investment was kept separate from circuit equipment investment, the two categories being developed in two runs through the same procedures. When circuit equipment investment has been computed, it is split between Ac count 221-57C (circuit equipment) and Account 221-67C (radio) investment. This separation relies on percentages exogenous to the analysis. Breaking down the investment total in this manner permits final output to be summarized by accounting categories rather than by plant categories.

Submodule 5: Independent Company settlements. There are over 10,000 telephone exchanges in the United States operated by Independent Telephone Companies. Each of these exchanges is connected directly or indirectly to the Bell System long distance network. "Settlements" procedures are required for dividing between Bell and Independents the revenues from traffic originating and terminating within Independent Company territories and interexchanged with the Bell System.

The Associated Companies maintain separate contracts with each of the Independent Companies with which they connect. These con tracts provide, among other things, for revenue divisions either on an individual cost study basis or by use of nationwide average settlement schedules. The average schedules are based on cost studies of a representative sample of Independent Company operations.

Within the model, the effects of the Independent Company settlements have to be determined prior to the final comparison of long-run incremental costs and net revenues.

Average schedule settlements are used, obtained through a study run by the U. S. Independent Telephone Association (USITA). This study, performed every two or three years, provides a specific cost per message for all companies, by function as follows:

(a) "A" Function — Revenue Accounting and Commercial activities, etc.

(b) "B-OH" Function — operator handling of calls "B-AT" Function — automatic ticketing of calls

(c) "Category 3 and Line-Haul" Function — use of toll switching equipment and circuits

A factor (cost per message) is applied to the Independent Company messages, to determine the settlement. For "A" and "B-OH" functions, there are different costs per message for each of 12 average revenue-per-message (ARPM) bands. For "B-AT" functions, a uniform cost per message is used. A ratio is applied to the total "A" function cost to derive the cost for the "Category 3 and Line-Haul" functions.

Data. The variables of interest are:

(1) *Exogenous variables.* Module-exogenous data describing the incremental demand in terms of messages, message minutes, and revenues are required. External data giving the distribution of average revenue per message (describing the relationship of Bell to non-Bell companies) and the USITA cost per message used in the revenue settlement procedure are also used in this module.

(2) *Endogenous variables.* Revenue settlements to Independent Companies are the output variables.

Procedure. The demand increments (expressed as messages, message minutes, and revenues) are input to this module, and an average revenue per message computed for each of the basic mileage bands used in the settlement procedures. The demand increment comprises traffic handled by both Bell and independent companies. To estimate the non-Bell portion of messages for settlement purposes, the distribution of average revenues per message (ARPM) is used. The USITA costs per message are applied to the estimates of non-Bell messages in each ARPM band.

The output is the estimate of incremental Independent Company settlements resulting from the changes in demand.

Submodule 6: Other additional costs and expenses. The following additional incremental costs are computed as functions of previously derived costs (with and without separations adjustments):

Account	Title
100.2	Telephone Plant Under Construction
100.3	Property Held for Future Telephone Use
101.1	Investments in Affiliated Companies
113–121	Cash, Working Funds, and Receivables
122	Material and Supplies
313.02	Interest Charged to Construction
305	Operating Taxes — Federal
	Gross Revenues
	Social Security
	State and Local
	Return on Investment
	Depreciation Reserve

1. *Telephone Plant Under Construction (Account 100.2).* This account represents the investment in telephone plant, other than station apparatus, that is not completely ready for service. It includes interest paid during construction, taxes paid during construction, and all other elements of cost connected with such construction work. When the plant is completed and ready for service, it is charged to the appropriate telephone (100.1) plant accounts. There are 15 sub-accounts which together comprise incremental investment: land (211), buildings (212), 7 categories of central office equipment (221), station apparatus (231), station connections (232), large private branch exchanges (234), outside plant (combined Accounts 241 through 244), and general items (Accounts 261 and 264). Factors were developed by relating the incremental investments in Account 100.2 to Account 100.1. The following general procedure is used to apply the factors within the model:

$$\Delta TP(X)_{100.2} = \sum_i \left[R(X)_i * \Delta I_i\right],$$

where

$\Delta TP(X)_{100.2}$ = change in telephone plant under construction,

$R(X)_i$ = the ratio of the incremental investments for a study period for the ith sub-account of Account 100.2 to Account 100.1 (i varying from 1 through 15), and

ΔI_i = the total annualized incremental investment in the ith sub-account of Account 100.1 derived from the model.

2. Δ *Property Held for Future Telephone Use (Account 100.3).* This includes the incremental cost of property (other than station apparatus) held for imminent use in telephone service (under a definite plan). The relationship between incremental changes in Account 100.3 and incremental changes in Account 211 (land) was developed through statistical analysis. The result obtained is then used to develop estimates of incremental amounts assigned to Account 100.3 as a result of a demand change, as follows:

$$\Delta I_{(100.3)} = \Delta I_{(211)} * R_{(100.3)}.$$

where

$$\Delta I_{(100.3)} = \text{Annualized change in cost of property held for future telephone use,}$$
$$R_{(100.3)} = \text{Ratio of incremental increase in 100.3 to Account 211, and}$$
$$\Delta I_{(211)} = \Delta \text{ annualized investment associated with Account 211.}$$

3. Δ *Cash, Working Funds (Accounts 113 through 121).* The relationship between cash, working funds, and investment in Account 100.1 (telephone plant in service) was developed by summing Accounts 113 through 121 inclusive, and studying them by applying statistical analysis techniques over time. This relationship is then used to estimate the incremental cash working capital necessary for providing incremental amounts of service. The accounts studied are:

113 — Cash — Current Funds Available for Use
114 — Special Cash Deposits
115 — Working Funds
116 — Temporary Cash Investments
117 — Notes Receivable from Affiliated Companies
118 — Due from Agents
120 — Accounts Receivable
121 — Interest and Dividends Receivable

The procedure for applying the resulting ratio is:

$$\Delta CWC = R_{(CWC)} * \Delta I_{(100.1)}$$

where

$$\Delta CWC = \text{Change in cash, working funds,}$$
$$R_{(CWC)} = \text{Ratio of cash, working funds to Account 100.1, and}$$
$$\Delta I_{(100.1)} = \Delta \text{ annual plant investment (Account 100.1).}$$

4. *Materials and Supplies (Account 122).* This includes the cost of unapplied material and supplies, tools, fuel, stationery, directory and paper stock, and other supplies, and material and articles of the company in process of manufacture for supply stock. Incremental investment in such items is estimated as a function of the incremental change in the outside plant accounts derived in the previous parts of the capital investment modules, by studying the relationship of incremental changes in Account 122 to Accounts 241 through 244 for a five-year period. The following procedure is used to estimate incremental costs for this account:

$$\Delta I_{(122)} = \Delta I_{(241-244)} * R_{(122)}$$

where

$$\Delta I_{(122)} = \text{Estimated incremental change in material and supplies,}$$
$$\Delta I_{(241-244)} = \text{Estimated incremental annual investment in outside plant accounts (241-244), and}$$
$$R_{122)} = \text{Ratio of incremental investment for Account 122 to incremental change in Accounts 241-244.}$$

5. *Interest Charged to Construction (Account 313.02).* This interest is the amount charged to the telephone plant accounts on funds expended for construction purposes. Estimates are obtained by applying a factor (derived exogenously) to the incremental amount estimated for Account 100.2 (telephone plant under construction). Thus,

$$\Delta A_{(313.02)} = F_{(313.02)} * \Delta TP(X)_{(100.2)},$$

where

$$\begin{aligned} \Delta A(313.02) &= \text{Interest charged to construction (Account 313.02)}, \\ F_{(313.02)} &= \text{Factor derived exogenously, and} \\ \Delta TP(X)_{(100.2)} &= \text{Telephone plant under construction (Account 100.2).} \end{aligned}$$

6. Δ *Investment in Affiliated Companies (Account 101.1).* Estimates of the incremental investment in securities issued or assumed by affiliated companies (other than securities held in special funds or as temporary cash investments) are derived as follows:

$$\Delta I_{(101.1)} = F_{(101.1))} * \Delta I_{(100.1)(A)},$$

where

$$\begin{aligned} \Delta I_{(101.1)} &= \text{Incremental investment in Account 101.1}, \\ F_{(101.1)} &= \text{Factor derived exogenously, and} \\ \Delta I_{(100.1)(A)} &= \text{Incremental annualized investment in telephone plant in service} \\ &\quad \text{(Account 100.1).} \end{aligned}$$

7. Δ *Operating taxes (Account 305).* The incremental effects on Federal, state, county, and municipal taxes relating to telephone plant, operations, and privileges for the test year are computed in four separate procedures.

(a) Δ Social Security Taxes

$$\Delta T_{SS} = \sum(\text{Wages}) * R_{ss},$$

where

Wages = sum of the following items:

Maintenance = $\sum([\text{Factors}_M^{(32)}$ * amount estimated for each account in maintenance expense module],

Commercial wage estimate,

Traffic = $[\text{Factor}_T^{(32)}$ * traffic expense estimate],

Revenue Accounting = $[\text{Factor}_{RA}\ ^{32}$ * accounting expense estimate],

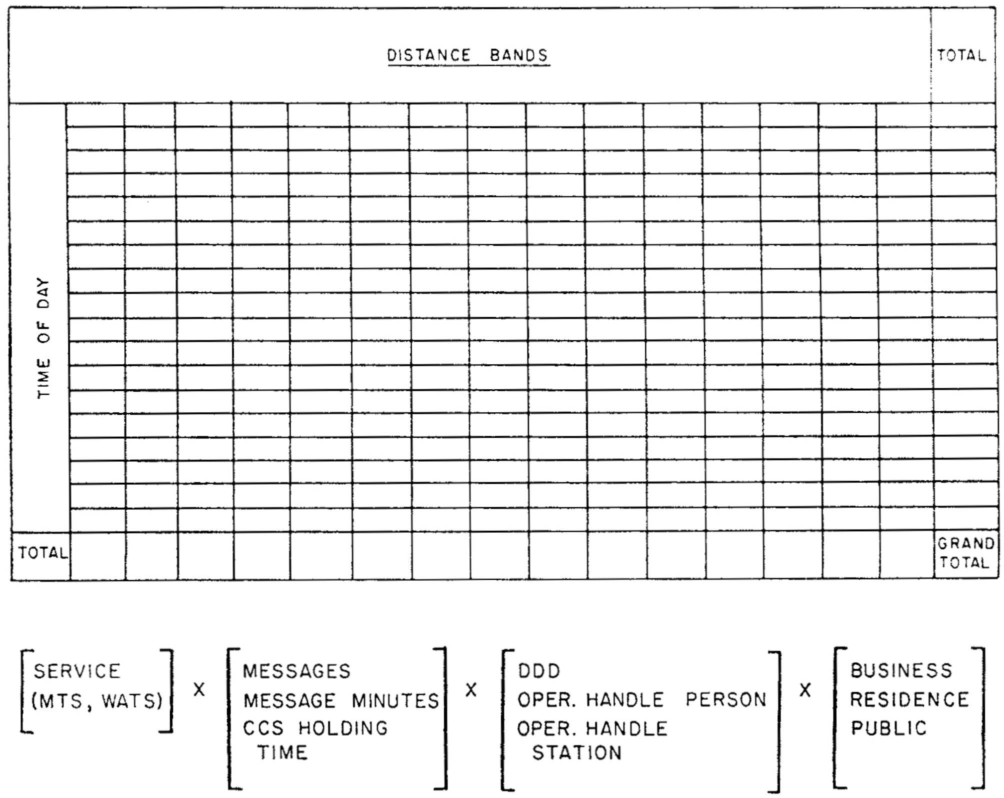
[32] These factors were developed by a special study using the last two years of data and estimating the ratio of wages to total expenses.

Other general office salaries, and

$$R_{ss} = \text{Tax rate (exogenous).}$$

TABLE 16

EXAMPLE OF DEMAND OUTPUT



(b) Δ State and Local Taxes[33]

$$T_{SL} = R_{SL}\left[\Delta REV - \Delta OPEXP\right],$$

where

$\Delta REV - \Delta OPEXP = \Delta$Net operating revenues, and

$R_{SL} =$ State & local aggregate tax(rate computed as an exogenous variable).

(c) Δ Federal Income Tax (Account 305)

$$\Delta T_{FI} = R_{FI}\left[(\Delta\text{Operating Revenues} - \Delta\text{ Operating Expense}) - \Delta\text{Other Tax}\right],$$

where

Δ Operating Revenues = Incremental revenues computed in earlier demand modules,

[33] Franchise tax was included as part of the estimate in General Expenses.

FIGURE 52

EXAMPLE OF TRAFFIC OUTPUT

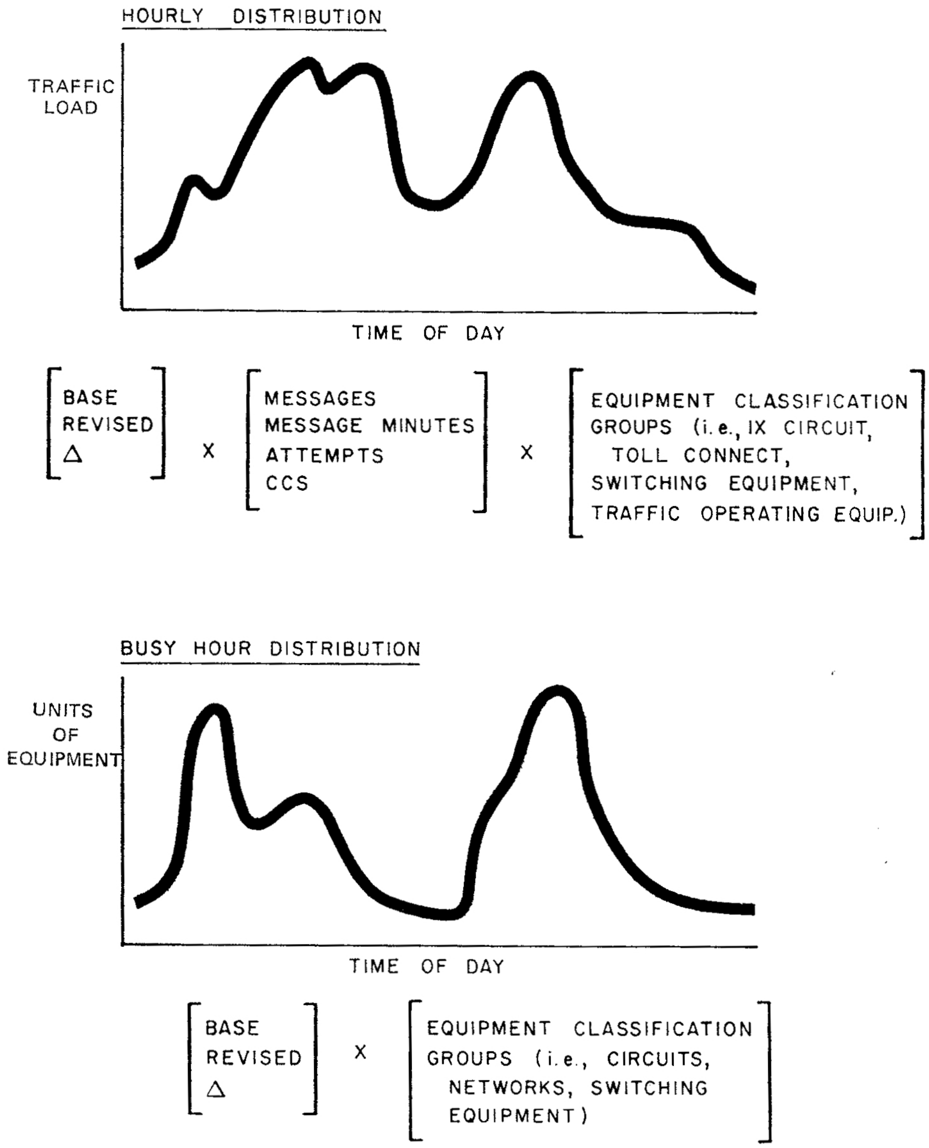
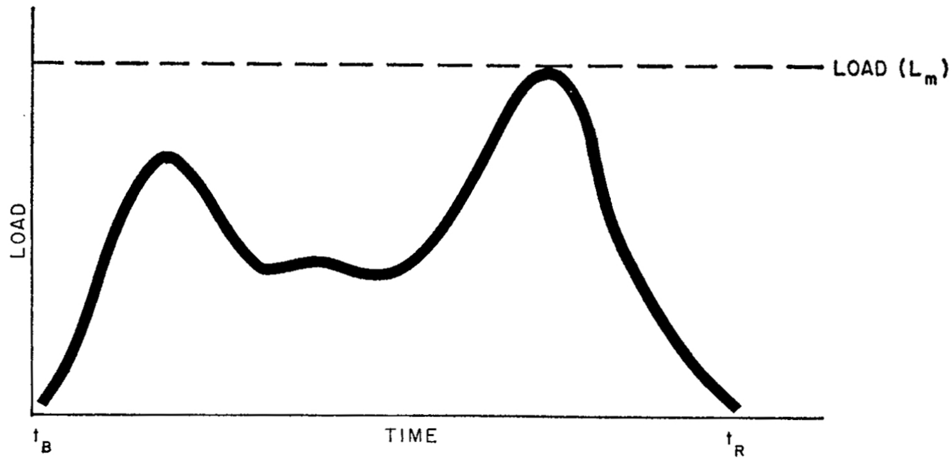


FIGURE 53

EXAMPLE OF MEASURES OF UTILIZATION OUTPUT



$t_B = t_{BASE}$, $t_R = t_{REVISED}$ } TIME PERIODS

L_m CAN BE DEFINED AS CAPACITY OR PRESENT UTILIZATION.

$$\text{UTILIZATION} = \frac{1}{L_m(t_R - t_B)} \int_{t_B}^{t_R} L(t)\,dt$$

(BY EQUIPMENT GROUPS, i. e., CIRCUITS, NETWORKS, SWITCHERS, TRAFFIC OFFICE.)

Δ Operating Expense = Annualized incremental expenses (including annualized amounts for capital investments), and

R_{FI} = Tax rate (derived exogenous to analysis).

(d) Gross Revenues Taxes

$$\Delta T_{GR} = R_{GR}[\Delta \text{ Revenues Adjusted for Settlement}],$$

where

R_{GR} = Tax rate (derived exogenous to analysis), and

Δ Revenues Adjusted for Settlement = Output of Module 16, Submodule 5.

8. Δ *Return on investment.* A rate of return is now computed based on annualized incremental net investment, to estimate the average annual return on investment. This return is used in the determination of incremental revenue requirements:

$$\Delta\text{Return} = (\phi)\left[\Delta V' - \Delta DRES\right]$$

where

$$\begin{aligned}\phi &= \text{Factor for estimated rate of return on net investment,}\\ V' &= \text{ capital investment derived in preceding parts of analysis, and}\\ \Delta DRES &= \Delta \text{ reserve estimates to adjust plant investment to an average net}\\ &\quad\ \text{investment for rate of return calculation.}\end{aligned}$$

9. Δ *Depreciation reserve.* Fourteen factors were developed for computing depreciation reserve, to be applied to 14 sub-accounts of Account 100.1. These factors are based on forward-looking average reserve ratios. The results, summed over the 14 sub-accounts, give total reserve figures. Accelerated depreciation using a declining balance method is applied to radio and coaxial hi-frequency line-haul book costs. The following designations are the major accounting codes and refer to various categories of plant:

Account	Item
211	Land
212	Buildings
17C	Manual central office equipment
27C	Panel central office equipment
37C	Step-by-step central office equipment
47C	Crossbar central office equipment
57C	Circuit central office equipment
67C	Radio central office equipment
77C	Electronic central office equipment
231	Station apparatus
232	Station connections
234	Large private branch exchanges
OSP	Outside plant
General Items	Related expense areas

Then

$$\Delta DRES = \sum_i \delta_i \Delta I_i,$$

where

$$\begin{aligned}\Delta DRES &= \text{Depreciation reserve adjustment,}\\ \delta_i &= \text{Depreciation reserve ratio for Account i, and}\\ \Delta I_i &= \text{Incremental investment in Account i.}\end{aligned}$$

Module 17: Reports. In this module, the derived information is summarized and the data output in various graphic and tabular formats. The following output, Levels 1 and 2, is typical of the aggregate results. Level 3 output consists of computer graphics and tables which permit deeper exploration of the effects and sensitivities of the price-cost relationships. Based on Level 3 results, the analyst may design additional runs of the model to test further interactions among the demand and supply aspects of the business.

Table 16 and Figures 52 and 53 are illustrative of the demand, traffic distribution, and utilization outputs, respectively, which are produced by the demand, Traffic expense and board investment, and circuit and switching investment modules.

Level 1 Output

Message Toll Telephone Analysis
Summary of Δ Annual Costs

Investment	________
Operating Expenses and Taxes	________
Federal Income Taxes	________
Amount for Return	________
Annualized Incremental Costs	________
Total Annual Revenues	________
Settlement adjustment	________
Total Annual Operating Revenues	________

(A) Reflecting separations
(B) Not reflecting separations

Summary of Δ Capital Investment Data

Outside Plant — Interexchange	________
Central Office Circuit Equipment — Interexchange	________
Central Office Switching Equipment — Interexchange	________
Central Office Radio Equipment	________
Station Equipment	________
Land	________
Buildings	________
General Items — Vehicles and Other Work Equipment, Furniture and Office Equipment, etc.	________
Other Additional Investment — Plant Under Construction, Cash Working Capital, Materials and Supplies, Miscellaneous Investment	________

Summary of Data on Δ Annual Operating Expenses and Taxes

Expenses for Maintenance (except Testing)	________
Depreciation, Property Taxes, General Departments, Advertising and Miscellaneous	________
Revenue Accounting and Commercial Expenses	________
Traffic Expenses	________
Testing Expenses	________
Gross Receipts Taxes	________
Other State and Local Taxes	________
Less Other Income — Interest During Construction, Interest on Temporary Investments, etc.	________
Total Operating Expenses and Taxes	________

Level 2 Output

Message Toll Telephone Analysis Listing of Standard Accounts[34]

	Investments	
Total	Sum Investments 100.1 through 100.3	________
100.1	Telephone Plant in Service	________
100.2	Telephone Plant Under Construction	________
100.3	Property Held for Future Telephone Use	________
	Current Assets	
Total	Cash Working Capital (113 through 121)	________
113	Cash	Not Estimated as Separate Accounts
114	Special Cash Deposits	
115	Working Funds	
116	Temporary Cash Investments	
117.1	Notes Receivable from Affiliated Cos.	
117.2	Other Notes Receivable	
118	Due from Customers and Agents	
121	Interest and Dividends Receivable	
122	Materials and Supplies	________
	Plant Accounts	
211	Land	________
212	Buildings	________

[34] By Account Number (column on left) and Account Title (column in center).

261	Furniture and Office Equipment	________
264	Vehicles and Other Work Equipment	________
221	Central Office Equipment	________
221–17C	Manual	________
221–37C	Step-by-Step	________
221–47C	Crossbar	________
221–57C	Circuit	________
221–67C	Radio	________
221–77C	Electronic	________
231	Station Apparatus	________
231–01	Teletypewriter	________
231–02	Telephone and Miscellaneous	________
231–03	Radio	________
232	Station Connections	________
232–02	Telephone and Miscellaneous	________
232–03	Radio	________
234	Large Private Branch Exchanges	________
Total	Sum Plant Accounts 241 through 244	________
241	Pole Lines	Not Estimated as Separate Accounts
242	Cable	
243	Aerial Wire	
244	Underground Conduit	

Income Accounts

305	Operating Taxes	________
305 B	Federal Income	________
305 C	State Income and Local	________
305 F	Social Security (in Gen. Off. Sal. & Exp.)	________
305 E	Ad valorem (in Gen. Exp. Items)	________
305 G	Gross Revenue	________
Total	Sum Income Accounts 312 through 316	________
312	Dividend Income	________
313	Interest Income	________
313–01	Interest Earned	________
313–02	Interest Charged Construction	________

Revenue Accounts

Total	Sum Accounts 510 to 516	________
510	Message Tolls	________
510–01	Telephone and Miscellaneous	________
510–02	Teletypewriter	________
511	Wide Area Toll Services	________

Maintenance Expense

601	Ordinary Repairs	______
602	Outside Plant Maintenance	______
603	Test Desk Work	______
604	Repairs of Central Office Equipment	______
17R	Manual	______
37R	Step-by-Step	______
47R	Crossbar	______
57R+M	Circuit	______
67R+M	Radio	______
77R	Electronic	______
604–04	Shop Repairs and Salvage Adjustment	______
604–06	House Service	______
604–07	Rearrangements and Changes	______
37M	Step-by-Step	______
47M	Crossbar	______
606	Repairs of Buildings and Grounds	______

Traffic Expense

621	General Traffic Supervision	______
624	Operators Salaries (Accounts 624+627)	______
626	Rest and Lunch Rooms	______
630	Central Office House Service	______

Commercial Expense

640	General Commercial Administration	______
642	Advertising	______
643	Sales Expense	______
644	Connecting Company Relations	______
645	Local Commercial Operations	______
648	Public Telephone Commissions	______

General Office Salaries and Expenses

Total	Sum Accounts 661 through 665	______
661	Executive Department	Not Estimated as Separate Accounts
662	Accounting Dept. (Other than Revenue Accounting)	
663	Treasury Department	
664	Law Department	
665	Information & General Secretary	______
662	Revenue Accounting	______

Other Operating Expenses

Total	Sum Accounts 668 through 677 plus 66509	————
668	Insurance	Not Estimated as Separate Accounts
669	Accidents and Damages	
671	Operating Rents	
672	Relief and Pensions	

Level 3 Output

Graphs and charts showing detailed analysis of changes in requirements:

Peak Loads Facilities Capital Investments Plant Utilization	By Major Plant Components[39]
Labor — manpower wages Operating Expenses etc.	By Major Operating Entities

[39] Example of Δ output by plant component (circuit facilities):

Classification

$$\begin{Bmatrix}\text{Interexchange}\\ \text{Toll Connect}\\ \text{Exchange}\\ \text{Special-WATS RX KCT-2}\end{Bmatrix} \times \begin{Bmatrix}\Delta\ \text{Route Miles}\\ \Delta\ \text{Circuits}\\ \Delta\ \text{Facilities by Type}\\ \Delta\ \text{Investment Costs}\end{Bmatrix} \times \begin{Bmatrix}\text{Account}\\ \text{Classification}\end{Bmatrix}$$

8. Computing and programming requirements

The requirements of this model, in terms of hardware, software, and staff, were extremely demanding. This was true due to the dynamic and complex nature of the model development and the exceptionally large scale of data processing. A substantial range of hardware and software systems were investigated in order to establish that sufficiently effective facilities would be available. After the fact, the choice seems to have been highly satisfactory. This is evident from the fact that the computing system development and data analysis tasks were completed essentially on schedule, and that it was found possible to reduce the complex set of programs to a production sequence which is routinely run for on-going analyses. Even more important, the model continues to be the subject of new extensions and further analysis capabilities as new data become available and new demands are recognized. Thus, the essential dynamic nature of the development has been preserved in the end (but not final) product.

It is also clear in retrospect that, regardless of the design and equipment choices, the expertise and dedication of the small group of programming staff were absolutely essential. This group of talented people, with the highest quality of facilities that could be provided, produced a truly remarkable result. In general, there is more to be learned from the principles which were applied in this aspect of the effort than from the specifics of the choices made at any particular moment in time. Technology and other factors are rapidly changing, so that such choices must be frequently reviewed, and prior commitments provide an uncertain guide. It is hoped that the guiding principles are more stable.

General design. The three basic factors which contributed to the general computer facility design were:

1. System accessibility for programmers and analysts,
2. System reliability, and
3. Substantial processing capacity, particularly for random access and for tape handling of very large files.

These factors led to a general system design which was highly modular, substantially self-documenting, entirely accessible from interactive terminals, and based on efficient direct I/0 techniques. While modularity of each component within each of the sub-modules was stressed, standard utility functions were also provided — and their use encouraged. This core of utilities, along with specified standards for interchange of data between modules, helped provide for effective data flow through the system, and for consistent usage of the data.

It was decided that an effective permanent file system would be essential. More specifically, a file system was required such that the programmers and analysts would not need to concern themselves with any general maintenance functions concerning any information

stored within the computer system. In addition, the file system should be sufficiently well organized so that it would be convenient for program and documentation storage while providing — within the same framework — highly efficient random access usage for very large dynamic data bases. These requirements placed very severe demands on any projected file system.

The other equally important computer facility requirement was terminal access. It was essential that the terminal access system be an integral part of the operating system, providing full access to all system facilities. This would enable system designers to carry out efficient development of large-scale program components, and would provide for effective ongoing analyses of "production" (computing) runs of the model. In addition, the terminal facilities should provide for convenient file manipulation and editing. These rather severe requirements grew out of the decision that for maximum benefit to be gained both in the development and in the uses of the model, all model testing and production use should be run from terminals. This decision has been carried out, and there is no card input to the system in the course of either model development or use. While during production runs, most of the steps in the analysis are run as batch jobs, however the runs are set up and monitored from terminals. During development, many of the model steps are executed directly at terminals in order to obtain direct interactive response.

It was considered essential for this approach that the operating system not distinguish between batch and terminal operation in terms of restrictions on executing programs, or in terms of available resources or system command language. It appears that the approach as outlined resulted in something like an order of magnitude increase in the productivity of the programmers and analysts on the staff.

Equipment requirements and choices. The main hardware systems available for meeting these requirements were the CDC 6400, the GE 635, the IBM 360/65/67, and the UNIVAC 1108. At the start of the effort, related work was being done on IBM 360's and GE 635's, as well as substantial data generation on IBM 7070's. Since direct first hand information was available on the IBM and GE systems, investigation was concentrated on the CDC 6400 and the UNIVAC 1108.

CDC had to be eliminated due to the lack of facilities for handling magnetic tape. Some tapes were entirely unreadable on CDC equipment, and the general reliability was unacceptably low. The GE 635 was eliminated for very similar reasons.

This left the IBM 360's and the UNIVAC 1108. The high-quality interactive facilities on the UNIVAC system and the lack of any such facilities on any 360 except the 360/67 meant that the choice was basically between the IBM 360/67 (using TSS as its Operating System) and the UNIVAC 1108 (using the 1100 EXEC Operating System). TSS terminal facilities were very good in some respects, but the flexibility of use and management of batch processing presented some problems. The TSS support of memory paging was conceptually excellent, but not so well supported in the hardware. The lack of a virtual processor concept for the support of multiple processors was a comparative defect, even in a unit processor environment. It did not appear that the 360/67 could provide the throughput

or file reliability which were required. The UNIVAC 1100 EXEC terminal facility design was excellent in terms of uniformity of batch and terminal processing, access to all system facilities, and ease of terminal use. The *implementation* of terminal facilities was, however, far from complete in either system, and both suffered significantly from their incomplete status and from oversights in what had been done. Since the 1100 EXEC was, and is, the primary Operating System available from UNIVAC, whereas TSS was a relatively minor development in IBM (since discontinued), it was felt that there would be continuing improvement in the UNIVAC implementation.

In terms of its current state, there were considerable doubts about the 1100 EXEC software system stability and system file maintenance facilities. However, detailed study led to the conclusion that the UNIVAC 1100 file system could be made to meet the project's needs. With these reservations, therefore, the UNIVAC system was selected.

However, some preliminary effort was still needed for extending the available system facilities. Most of this effort could be carried out without any modification to the Operating System. Thus, the problem of conflict or duplication of effort by the project team and the UNIVAC system development people could be avoided. The areas requiring improvement were chiefly file maintenance, system stability, and improvement in text editing facilities. These efforts were completed successfully and the UNIVAC 1100 system itself evolved in a highly successful and effective way during this period.

General software development. The first choice required in the course of the general software development effort was that of the main programming language. It was fairly obvious that there was no practical choice but to accept FORTRAN. However, as in other similar situations, the more important choice was in the kind of FORTRAN programming standard which would be required or encouraged. The effectiveness of FORTRAN as a language for large programming projects depends very much on how the language is to be used, and on the general facilities available to support the language.

There were three important aspects to this particular use of FORTRAN which improved its effectiveness. First, the maintenance of FORTRAN source code together with its documentation in online files was encouraged. This meant that there was a tendency to develop, debug, and document the programs in an integrated, incremental manner. Second, the use of the "macro" facilities provided in UNIVAC FORTRAN was encouraged. The principal benefit of this was to remove the need to duplicate elements of program text, which greatly facilitated updating of shared information and avoided the possibility that duplicates would be updated in an inconsistent way. This sharing of information at the symbolic program level was clearly essential to the smooth flow of program development. (If this facility had not been available directly within UNIVAC FORTRAN, one of the generally available FORTRAN macro preprocessors would have been used instead.) Third, libraries of utility routines were developed which could be called from FORTRAN to provide a common base for many widely used functions. In this area, the basic tools already existed in the UNIVAC system; however, considerable effort had to be spent in development of the actual library routines. This effort proved to be well worthwhile since, as at the level of symbolic

program text, the removal of duplication of information through the collection of common requirements led to very great improvements in both efficiency of program development and consistency of developed programs.

Module design and interface standards. Within the general guidelines indicated above, specific design requirements were established for each module as well as specific interface requirements for each interconnection between modules. The module design requirements varied according to the nature of each module. For modules which manipulated relatively small quantities of data, direct use of FORTRAN I/0 was permitted. Thus, many of the smaller modules could be written entirely in FORTRAN with very minor use of system dependent utility functions. For larger modules, a system-dependent direct-access I/0 facility was used. This utility library facility was designed to provide the most efficient possible I/0, and to provide for the fullest possible debugging and error information. Essentially a table-driven direct I/0 system, it proved to upgrade I/0 efficiency in some cases by several orders of magnitude over conventional FORTRAN I/0.

Regardless of the I/0 technique selected for a given module, each was required to have a FORTRAN subroutine for reading and writing its data. This, coupled with suitable variable descriptors, meant that the data could be considered to be self-describing to each interconnecting module.

Finally, due to the very large number of parameters in the system, and their widespread use, a common parameter interchange standard was developed. The basis of this scheme was the organization of the sets of parameters into standard FORTRAN COMMON blocks. These COMMON blocks are maintained in a permanent file. This file is easily updated, displayed, and referenced by each module that requires reference to parameter values.

The intertoll circuit (IX) analysis module. The design of this module will be discussed in detail because it presented one of the major analysis problems, and because it requires a substantial portion of the total computing resources used in operating the entire model. In earlier sections [p. 85], the circuit requirements model and its underlying theory were described. Here, the computational aspects will be treated. The computational complexity of this module stems from the fact that network costs are a strongly non-linear function of network load. This point has already been discussed in general terms, but is repeated here to remind the reader of its key importance. By "strong" non-linearity, we mean forms of non-linearity which cannot be reasonably approximated, even locally, by any smooth function. This form of non-linearity in network characteristics stems mainly from the discontinuous effect on peak-load behavior of even small changes in the hourly distribution of point-to-point loads. The fact that equipment requirements are determined by peak-load conditions leads to the discontinuous relation between incremental cost and point to-point load changes.

Due to this strong non-linearity, no reliable aggregate functional description of IX system behavior has been developed. Thus it was necessary to construct an analysis which would compute system requirements based on the load behavior for each node in the system. In order to appreciate the computational requirements of this analysis, it should be

remembered that there are about 15,000 interconnecting links and about 1900 nodes in the IX system. (This is the census of all offices at the class 4 level and above.) The data which characterize a link in the network require about 4400 bytes of computer storage space.

Since the analysis must start from a change in the load on each link (induced by a change in prices or user behavior), the first requirement of the module is the management of load data for each of the links. The basic data classification for many of the analyses has been by:

1. 15 price-distinguished mileage bands,
2. 18 hours of the day (grouping midnight-6 A.M.), and
3. 4 classes of IX switched-network service (MTS-Interstate, MTS-Intrastate, WATS, and Other).

Thus, the basic load data for the links in the system comprise a file of about 72 million bytes. Obviously, during the flow of computation through the module, various aggregations of these basic arrays need to be computed and stored.

In addition, since the circuit requirements for a given load are represented by probability models of telephone traffic, the arithmetic computation requirements of the analysis are also substantial. Specifically, the computation of the number of circuits required for the original load and for the new load resulting from a change in point-to-point load must be carried out for each of the 15,000 links. Thus, all link-related computation must be carried out about 15,000 times whenever a new set of load changes is processed. Even fairly small changes in efficiency can, when multiplied by 15,000, have a substantial effect. (One second times 15,000 is about 4 hours, 10 minutes!)

An initial attempt at carrying out this analysis was developed using essentially sequential files. This required several hours of computer time for an incomplete analysis of network structure, and it was clearly not possible to extend the analysis to the entire hierarchy of networks nor to properly incorporate the analysis of overflow traffic. This initial version provided important insights into both computational and analysis requirements. The first version of the module which was integrated into the analysis system benefited very much from the experience of this preliminary version.

Due to the program running-time estimates which were obtained from the preliminary version and other calculation, it was obvious that there must be three main considerations:

1. Efficient random-access I/0 for files of link and network data were needed,
2. Efficient algorithms for evaluation of probabilities and parameters of the Poisson and Erlang-B probability distributions were needed, and
3. Facilities for concurrent computation and I/0 data transfers would have to be provided.

The intent of pursuing these points was to produce a sequence of sub-modules such that the incremental circuit requirements (by appropriate sub-classification) could be produced from the input of incremental link load requirements within a total elapsed computing time of a few hours. (This goal was based on the overall requirement that it be possible to carry

out a complete analysis through the entire system within one day.)

While the program structures were designed with this objective in mind based on the characteristics of the UNIVAC 1108 (the computer system which was first used), which has a memory cycle time of 750 ns, the computing was subsequently moved to a UNIVAC 1106 system which has memory exactly half as fast as the 1108. Present computational performance of the system on the 1106 is a complete analysis cycle in about 4 hours. The IX module is completed in about 2 hours. While the original timing estimates may appear now to have been somewhat conservative, much other evidence of similar computing suggested that they were wildly optimistic at the time.

An additional aspect of the operational considerations in the processing cycle of the system is that the entire system was required to be processed in an "open" environment. This means that the computing system is always open to general interactive terminal and development batch-processing work while it is also processing analysis cycles. This requirement was imposed in order to insure that the development effort be facilitated and that the analysis cycle processing be maintained in a sufficiently manageable form so that interactive monitoring, control, and partial analyses could be conveniently carried out. This has proved to be a key factor in the usefulness of the IX module and of the entire analysis system.

Technical considerations in the IX module. As must be obvious, some special attention was required in the IX module in order to achieve reasonable processing rates. Due to the complex nature of the module structure, the constraints imposed by the deadlines for operational use, and the essential dynamics of the model development, the main emphasis in the module design was on techniques which would yield efficiency gains of approximately an order of magnitude. Consequently, very little marginal efficiency tuning has been carried out.

The first important technical development was the construction of a direct random-access I/0 facility. This facility provides for direct access I/0 of arbitrary size records, and, specifically, provides for the reading or writing of multiple or partial records with one I/0 command.

Next, the logical structure of the various data files which the module requires was developed. In order to provide efficient referencing of data records, accessing techniques were chosen such that the records could be, under the most common conditions, referenced directly. This means that the program could read or write the record using just one I/0 request. Of particular importance were the following conditions:

1. The data (including such items as the link membership) for each node in the network should be directly accessible given the name of the node switching machine.
2. The data records for each link in a given network should be directly accessible given the list of member links provided in the data records for each node.
3. All link records should be sequentially accessible in a manner such that any multiple number of records could be read or written by each I/0 request. The link records must also, of course, be accessible from a list of link identifiers. This organization permits

relatively fast reading or writing of link records in a sequential mode, such as when summary statistics are being computed.

In order to permit simultaneous processing of separate tasks within one module, a simultaneous task control routine was developed. This was not very difficult, since the 1100 Operating System provides a very powerful multi-activity facility. It was only necessary to develop a FORTRAN interface to the system facilities and to provide some additional higher level interlock control logic. The main use of this facility has been to provide simultaneous processing of network load characteristic computations. These require both substantial computation and substantial input and output of data records. The IX sub-module, which is responsible for this computation, processes several networks in parallel. This permits effective overlapping of processor and I/0 use. On a multi-processor system, the program would also use several processors simultaneously. A general-purpose parallel processing facility was developed, and several components of the model benefited substantially by making use of this capability.

Another important constraint was that the size of the sub-modules should be reasonably small and should not depend on the number of links contained in the data base. The size of the sub-modules does depend on the number of networks, but due to the original size restriction it is easy to expand the capability of the module to handle even several times the number of networks as are currently present in the intertoll network census.

All of the tables and data records in the module are formatted through the use of a set of parameters which are held in one common element. These parameters are used throughout the module for referencing any table or data record entry. Thus, the tables and data records may be modified, for instance to include some new item of data, by simply inserting a new parameter. This capability extends, for example, to changing the number of types of service which may be kept in each network and link record. It proved to be extremely valuable since, as analyses developed, it was often found that additional information was needed or, occasionally, that information became irrelevant. Due to unusually powerful facilities in the 1100 system FORTRAN compiler, this flexibility could be maintained without any loss of efficiency at execution-time and without having to resort to the use of a FORTRAN preprocessor.

Algorithms used in the IX module. The IX analysis requires both substantial I/0 and substantial computation. Thus it was important to develop efficient algorithms for evaluation of the relevant distribution functions and their required inverses. However, it was also found that these algorithms needed to be quite robust, as they are required to converge over very wide ranges of values of some of the parameters.

It was originally thought that Newton-Raphson iterative approximation algorithms would be most efficient and effective. This turned out not to be the case for two reasons. First, the Newton-Raphson algorithms failed to converge under certain circumstances. These circumstances were fairly rare, but rare failures are a considerable nuisance when using the algorithm about 15,000 times in each run of the sub-module. In typical analyses, the algorithm would fail to converge only a few times. Some work was devoted to attempting to improve the convergence properties of the implementation of the algorithm. However, it became evident that this was not making the required improvement and was causing a decrease in efficiency.

For these reasons, a set of algorithms was written using the method of False Position. A good property of the method of False Position is that convergence is guaranteed. The apparently bad property is that convergence is linear, whereas convergence of the Newton-Raphson algorithm is quadratic. Thus, one would expect the False Position technique to be slower. However, it was found that by careful selection of the starting point for the algorithm the method of False Position could be made at least as fast as the Newton-Raphson code.

Finally, considerable effort was spent on the convergence tolerance, since this strongly affected efficiency for both algorithms. It was found useful to adjust the tolerance so that the required accuracy was attained as a function of the parameters of the distributions, rather than by simply taking a fixed tolerance.

Conclusions. The general conclusion that can be stated at this point is that the basic approach of integrated on-line computer use made possible the rapid development of an analytically and computationally complex model. While many of the details of choices were dictated by available data and equipment, and are thus of mainly historic interest, the strategy of high-level computer use may have more permanent meaning. Throughout the project, nearly all of the participants had direct contact with the computer system. Specific attempts were made to avoid making distinctions between programmers, analysts, or designers. This had a decisive effect on the integrity of the entire model system.

There is a tendency, sometimes explicitly stated, to treat computation, data analysis, and programming as tasks to be carried out by a relatively unskilled staff. While this idea has a rather long history, going back at least to Leibnitz, it seems to be founded on a major fallacy. In fact, on the contrary, a computing system should have as one of its primary design goals its accessibility to everyone, and especially to the most creative analysts and research workers, for whom it has been found to be particularly effective. It is hoped that some important steps have been taken toward realization of this goal during the project which this work describes.

9. Applications of the model

The model can be used in a number of operational modes to research price-demand-cost relationships pertaining to the major switched network services. Some of the principal applications are:

(1) To identify the important aspects of pricing and costing principles and their relationships.

(2) To provide guidance for making rate adjustments, as well as robust indications of the "best" directions to be taken, by estimating:

   (a) Total price-related incremental costs that are free of additive or linear bias, and

   (b) Marginal cost approximations.[40]

(3) To provide guidance in pinpointing sensitive or high-risk areas in pricing policy.

(4) To provide insight into price-cost interactions, including the effects of the separations policies.

(5) To monitor and systematize price changes.

Simplified examples of ways in which the model can be used for rate policy guidance and research will be illustrated. Since many of the current coefficients and assumptions used in the model are first approximations, and are currently being researched, these examples should be regarded as purely illustrative. The first shows the estimated total long-run incremental cost associated with a price change. The second shows estimates of unit incremental (or approximations to marginal) costs associated with given segments of a service offering.

To clarify these estimates, let us review briefly some of the earlier points made with respect to network service offerings. Table 4 (p. 43) summarizes the major services offered to the public. Within each major service classification there exists a nesting of specific rate categories. Figure 22 (p. 44) illustrates this nesting effect for MTS Interstate service. Note the number of categories which define this one service space alone, and in turn specify the price the customer pays. These are the levels at which marginal costs are relevant in evaluating the direction in which prices should move to accomplish specific pricing objectives.

Figure 23 (p. 50) illustrates the service space, which should be thought of as a vector; any demand change (ΔD) for a specific service or subclass of service will result in a change in this vector. Changes in the revenue/cost hyperplane were discussed in Section 6. Consider again the surface which would result if one sliced down the ordinate and out along one of the vectors created by the demand change. The incremental total costs and total revenues and the marginal costs can be viewed as illustrated in Figure 24 (p. 52); all are summarized in Table 17 (p. 196).

[40] These estimates are derived such that the various interactions in both demand and cost spaces can be evaluated.

The price-cost relationship. In this example the analysis is used to assist in the study of price-cost relationships associated with price changes. To simplify the example, a price change will be considered for only MTS-I service, operator-handled (OPH) person messages, originating between 5 p.m. and 11 p.m. for distances over 200 miles. The price of OPH station messages will be assumed to increase non linearly by approximately 5 percent for calls over 2000 miles. This is, highly simplified since, in the normal case, the prices for the various services and their sub-classifications may be changed simultaneously. Since the network serves all users on a joint and common basis, this requires handling all of the relevant interactions simultaneously. An essential aspect of the model is the capability to do this.

TABLE 17

MARGINAL	INCREMENTAL
PRODUCT = q	PRODUCT = "SERVICE"
COST = $C(q) + b$	COST = C (NETWORK)
REVENUE = $R(q)$	REVENUE = R (NETWORK)
MARGINAL REVENUE = $\frac{\delta R}{\delta q}$	INCREMENTAL REVENUE = $\frac{\Delta TR}{\Delta D} = R_2 - R_1$
MARGINAL COST = $\frac{\delta C}{\delta q}$	INCREMENTAL COST = $\frac{\Delta TC}{\Delta D} = C_2 - C_1$

Detailed output is available for studying and evaluating the effects of the price change. This includes summaries on changes in demand and revenue relationships; Δ traffic characteristics to assist in understanding the peak and off-peak characteristics; equipment changes that will be required over the planning period; etc. Only a few basic summary results will be shown to illustrate the analysis. Table 18 summarizes the estimated changes in total annual messages that result from a price change. The OPH station messages decrease overall, although some of the OPH station and DDD messages increase because of shifts in the *pattern* of customer usage. The overall effect is a decrease in messages, however, when the price of OPH station calls is raised.

TABLE 18

Δ ANNUAL MESSAGES

	WEEKDAY		SATURDAY		SUNDAY		TOTAL	
OPH PERSON	+ 1,500,000	+.006	+ 150,000	+.019	+ 250,000	+.018	+ 1,850,000	+.007
OPH STATION	−20,400,000	−.031	− 5,400,000	−.067	−9,000,000	−.090	−34,800,000	−.042
DDD	+13,200,000	+.007	+ 3,600,000	+.018	+ 5,300,000	+.023	+ 22,150,000	+.010
TOTAL	− 5,700,000	−.002	− 1,650,000	−.006	−3,450,000	−.010	− 10,800,000	−.003

Table 19 shows the estimates of the Δ change in total annual revenues resulting from the price change. Note that the model estimates that the increase in price of OPH station initial period results in a decrease in revenues in that category. The shift of messages to OPH person and DDD has resulted in an increase in these categories.

The net effect is an increase in revenues, because the price increase in the OPH station messages is applicable to the total OPH station base. This has the effect of partially offsetting the loss of revenues which is caused by the loss of messages in that category.

The information shown in Table 20 is a condensed summary of the Level 1 output.

By exercising the separations option, one can study the effects of the jurisdictional procedures.

The major intent of this example is to illustrate how the model can be used as a tool to study alternatives, by making iterative runs based on selected patterns of price changes which allow price-cost relationships to be explored. This exploration provides guidance on directions as to how prices can be changed in order to achieve corporate pricing objectives.

TABLE 19

Δ ANNUAL REVENUES

	WEEKDAY		SATURDAY		SUNDAY		TOTAL	
OPH PERSON	+ 6,000,000	+.008	+ 450,000	+.013	+ 950,000	+.023	+ 7,400,000	+.009
OPH STATION	− 18,100,000	−.014	−4,900,000	−.033	−12,850,000	−.054	−35,900,000	−.022
DDD	+ 31,100,000	+ .010	+ 6,600,000	+.024	+ 13,800,000	+.033	+ 51,600,000	+.014
TOTAL	+ 19,000,000	+.038	+ 2,150,000	+.005	+ 1,900,000	+.003	+ 23,100,000	+.004

TABLE 20

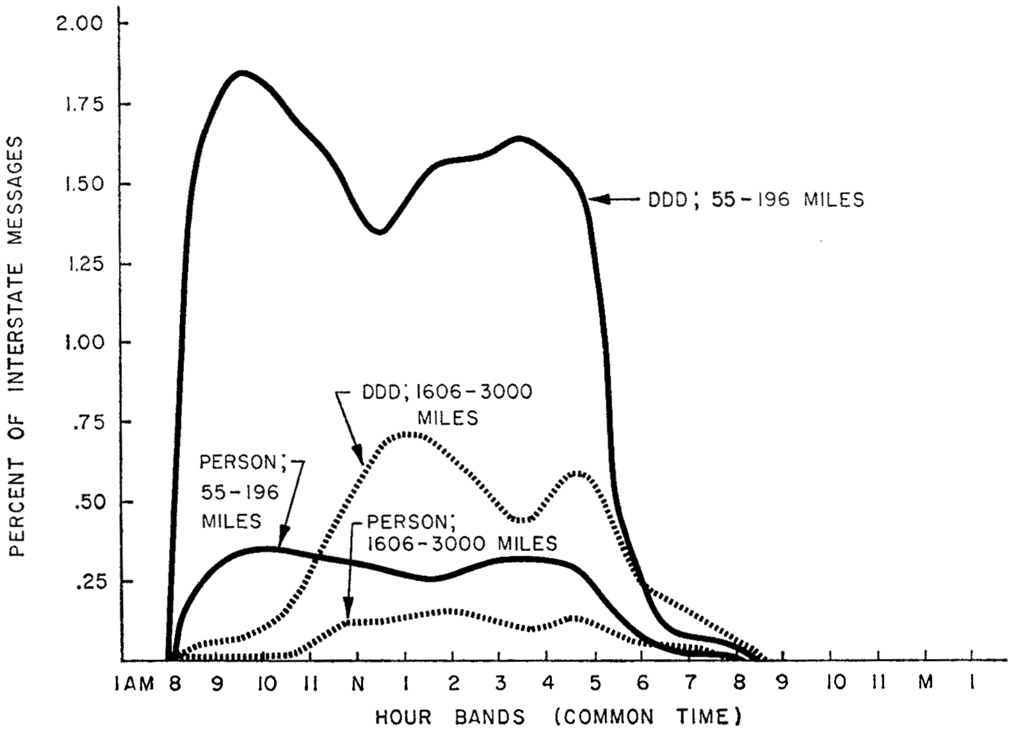
SUMMARY INFORMATION REVENUE REQUIREMENTS

LEVEL I OUTPUT (NOT REFLECTING SEPARATIONS ANALYSIS)

PLANT INVESTMENT (PRESENT WORTH TO BASE PERIOD)	THOUSANDS OF DOLLARS
OUTSIDE PLANT	−900
CENTRAL OFFICE CIRCUIT EQUIPMENT	−11,500
CENTRAL OFFICE SWITCHING EQUIPMENT	−2,100
CENTRAL OFFICE RADIO EQUIPMENT	−400
LAND	−200
BUILDING	−2,700
GENERAL ITEMS — VEHICLES AND OTHER WORK EQUIPMENT, FURNITURE, ETC.	−800
OTHER INVESTMENTS	−2,500
TOTAL	−21,000
ANNUAL OPERATING EXPENSES	
MAINTENANCE EXPENSES	−1,100
ACCOUNTING & COMMERCIAL EXPENSES	−500
TRAFFIC EXPENSES	−9,900
GENERAL DEPARTMENT EXPENSES AND ANNUALIZED INV. ANNUITY	−4,500
SUBTOTAL	−16,000
LESS OTHER INCOME (INTEREST DURING CONSTRUCTION)	−100
	−15,000
ESTIMATED RETURN ON INVESTMENT REQUIRED	−1,700
ESTIMATED EFFECTS ON ANNUALIZED Δ COSTS (REV. REQ.)	−16,700
ESTIMATED ANNUAL Δ REVENUE	23,100
ESTIMATED SETTLEMENTS	11,900
ESTIMATED ANNUAL OPERATING REVENUE	11,200

FIGURE 54

% CHANGE MESSAGES DUE TO +10% CHANGE IN LOAD 8 AM – 8:30 PM

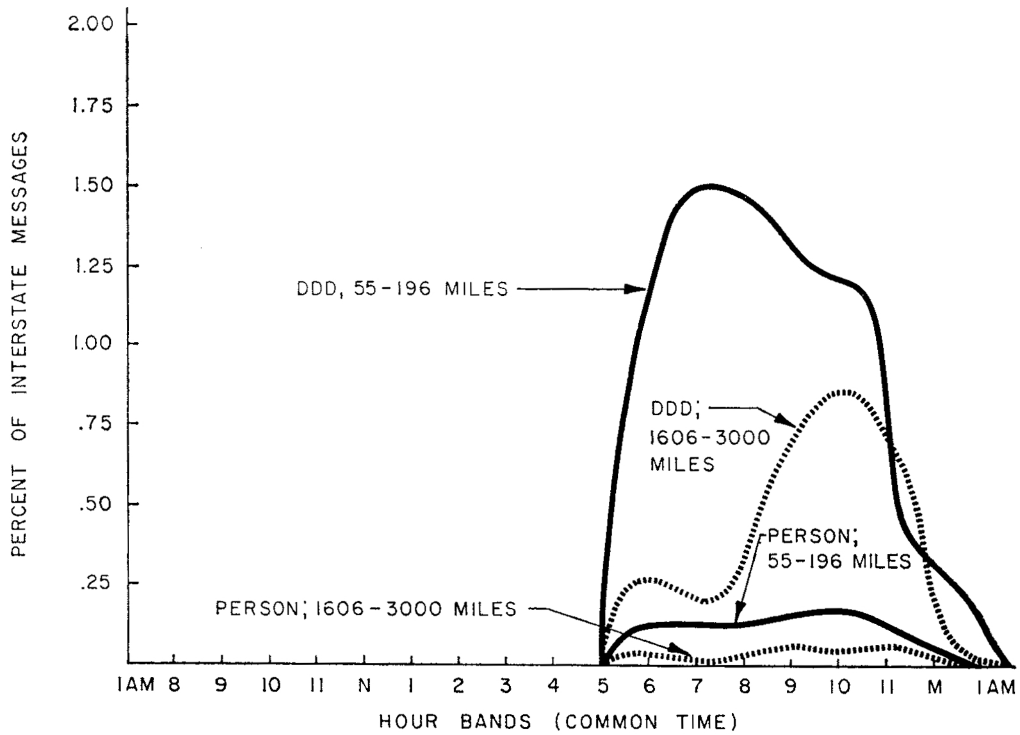


Estimates of marginal unit costs. Another operational mode can be used to derive approximations to marginal unit costs. In the next example, the change in demand will be attained by incrementing the demand resulting from a given set of prices by a percentage. This percentage is a critical value, since the unit costs are not independent of the size of the increment. It should be noted that this mode represents an entirely different use of the model, since in the computation of marginal costs there are no assumptions made about price changes. Also, the geographical characteristics of traffic and patterns of customer behavior, such as shifts in usage and length of conversation, are assumed unchanged for the test period.

Marginal costs are developed by analyzing the effects on investment cost and expense of incrementing demand in categories of interest, employing a statistical design. Iterative runs of the model are first made, to develop the base for estimating marginal costs. Then the relationships among marginal cost data are evaluated. In this example, a simple factorial design is employed in which the main factors studied are class of call (DDD, OPH person), time of day (8 a.m. to 5 p.m. 5 p.m. to 11 p.m.), and rate mileage (medium — 55 to 200 miles; long — 1600 to 3000 miles).

FIGURE 55

% CHANGE MESSAGES DUE TO +10% CHANGE IN LOAD 5 PM - 1AM



Only weekday MTS-interstate usage, in terms of messages, is studied in this experiment. If message minutes or some other units of demand were incremented, rather than messages, the unit costs developed (as well as their interrelationships) would be different from those shown here. Therefore, it is apparent that the unit of demand selected for study is in itself an important item for investigation. Furthermore, length of conversation, ratio of attempts to completed calls, elasticities, and other aspects of demand are all held constant in this case. Therefore, the results reflect only certain interactions in the cost space, neglecting those which occur in the demand space.

Each of the main factors is studied at two levels, which creates a 2 x 2 x 2 analysis. This involves eight runs of the model and allows insight to be gained as to the existence of second-order interactions.

TABLE 21

EFFECTS ON MESSAGE STATISTICS WHEN DEMAND INCREASES BY 10%

CLASS	DDD				PERSON OPERATOR			
TIME	8 AM – 5 PM		5 PM – 11 PM		8 AM – 5 PM		5 PM – 11 PM	
DISTANCE	MED	LONG	MED	LONG	MED	LONG	MED	LONG
△ ANNUAL COST PER MSG.	.30	.50	.30	1.40	1.30	1.50	1.30	2.60
△ ANNUAL OPER.* EXP. PER MSG.	.20	.30	.20	.50	1.00	1.20	1.00	1.70
△ INVESTMENT PER MSG.	.60	1.10	.80	5.40	1.90	2.10	2.10	6.40
△ REV. PER MSG.	1.20	3.40	1.20	2.60	1.90	5.90	1.90	5.00

This simple example is merely illustrative of the results which can be obtained through certain techniques; in more complex investigations, one would normally use different arrangements of the various factors and levels in order to cut down on the number of model runs required. When such arrangements are used, however, it is not possible to observe certain interactions which can be studied when the main factors are investigated separately at each level of interest.

Figures 54 and 55 show the percent change in total interstate load resulting from incrementing the usage categories selected. The percent change in *total* load on the network would be still smaller. However, an increment of 10 percent in the demand categories shown is not an in significant one, and the results are interesting to observe. Tables 21 and 22 are arrays showing the results in terms of messages and message minutes, respectively, when demand is increased by 10 percent. The results are given in the following categories:

(a) Annual cost,
(b) Investment, in terms of present-worth at the base period,
(c) Annual operating expense, and
(d) Annual revenue.

TABLE 22

EFFECTS ON MESSAGE-MINUTE STATISTICS WHEN DEMAND INCREASES BY 10%

CLASS	DDD				PERSON OPERATOR			
TIME	8 AM-5PM		5 PM-11 PM		8AM-5 PM		5PM-11PM	
DISTANCE	MED	LONG	MED	LONG	MED	LONG	MED	LONG
△ ANNUAL COST PER MSG-MIN	.05	.06	.04	.16	.22	.17	.14	.25
△ OPER EXPENSE PER MSG-MIN	.04	.04	.03	.08	.17	.14	.10	.15
△ INVESTMENT PER MSG-MIN	.13	.15	.10	.55	.34	.24	.27	.70
△ REV PER MSG-MIN	.23	.48	.15	.27	.33	.69	.25	.54

In evaluating the results, we will study the main effects first. These charts show the significance of class of call, time of day, and rate mileage (distance) on annualized cost and investment. In using this information as a basis for structuring pricing and costing policies, it is important to evaluate whether there are significant interactions among factors — that is, whether or not the average effect of a factor may be expected to differ at various levels of the other factors. If it does, then the factors can be said to interact; the *effect* of one factor depends on the *level* of another. The existence of a significant interaction would lead one to conclude that, in making price/cost decisions, the impact of a change on not only the primary factor but the secondary (interacting) factors should be seriously weighed.

Figures 56 through 60 illustrate graphically the factor interactions for this two-level simplified analysis. Lines plotted parallel to the abscissa imply no interaction among the factors involved. (Note that the lines are for graphic illustration, the two points for the two-way classification are arbitrarily positioned along the abscissa.) When the lines are not parallel to the abscissa, such as is the case for the long-haul vs time-of-day results (Figures 57-60, upper right graph), significant interactions are implied. Figure 56 summarizes the main effects, and the figures which follow show the relationship of the length-of-haul and time-of-day factors with respect to incremental cost (annualized) per message, investment per message, and operating expense per message. The effect on revenue per message, shown in Figure 60, of the interaction between length of haul and time of day is significant because

it reverses the pattern of the interactions shown in the following three graphs (Figures 57, 58 and 59). Here, revenues decrease later in the day, while Figures 57, 58 and 59 have shown that investment, costs, and expenses rise during the later period for long-haul messages.

FIGURE 56

MAIN EFFECTS

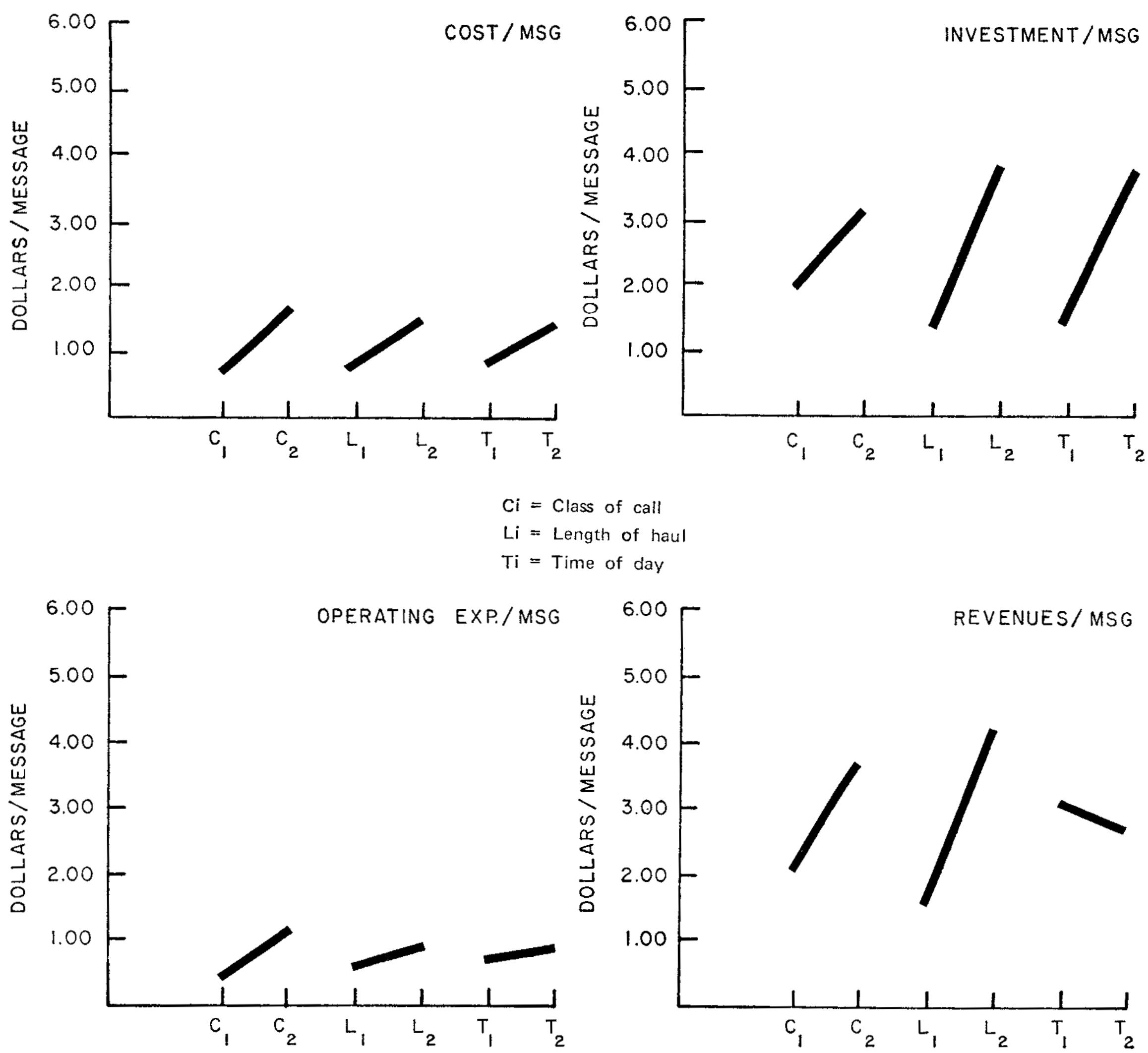


FIGURE 57

COST / MESSAGE, (2 X 2) INTERACTIONS

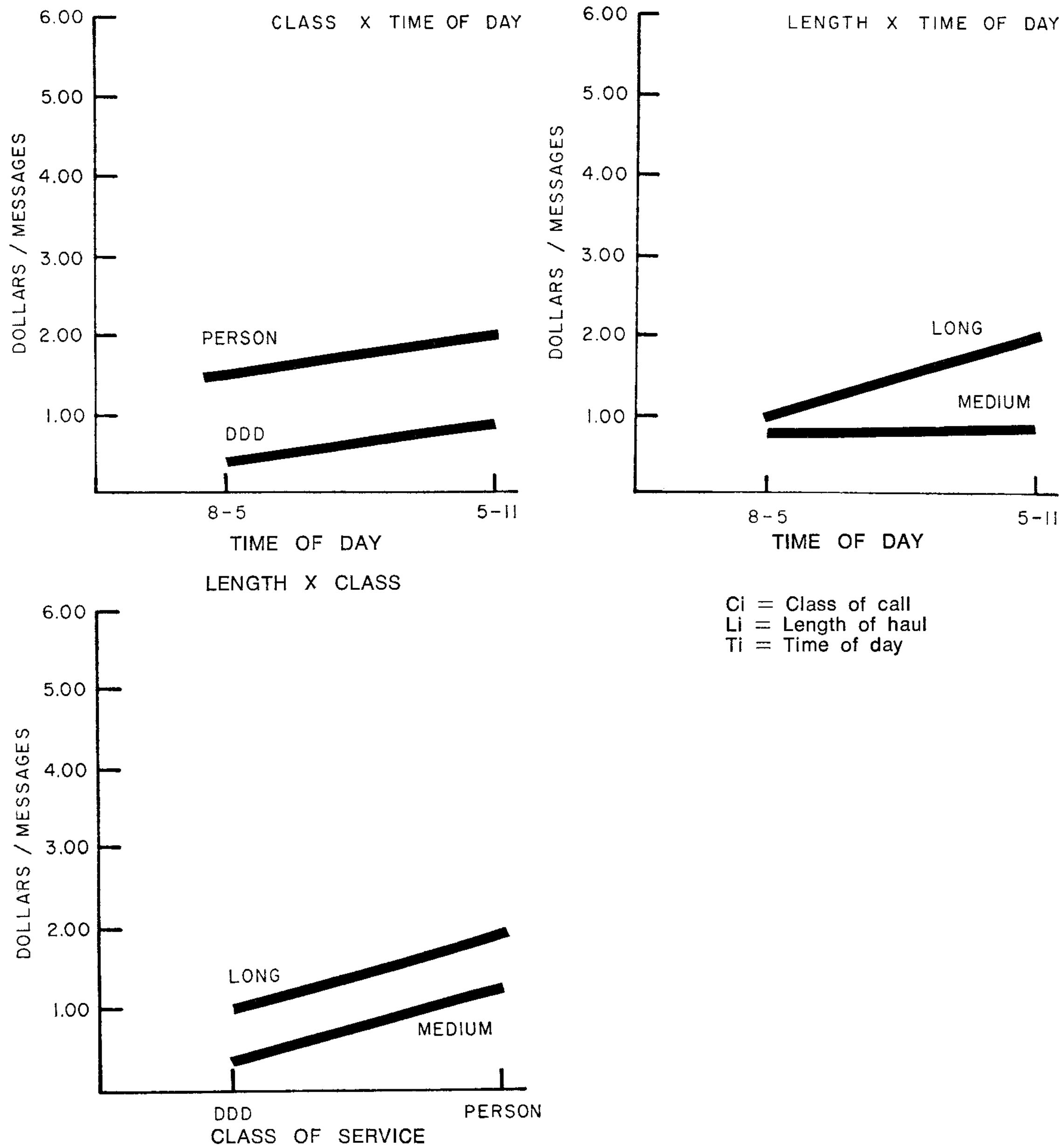


FIGURE 58

INVESTMENT/MESSAGE, (2 X 2) INTERACTIONS

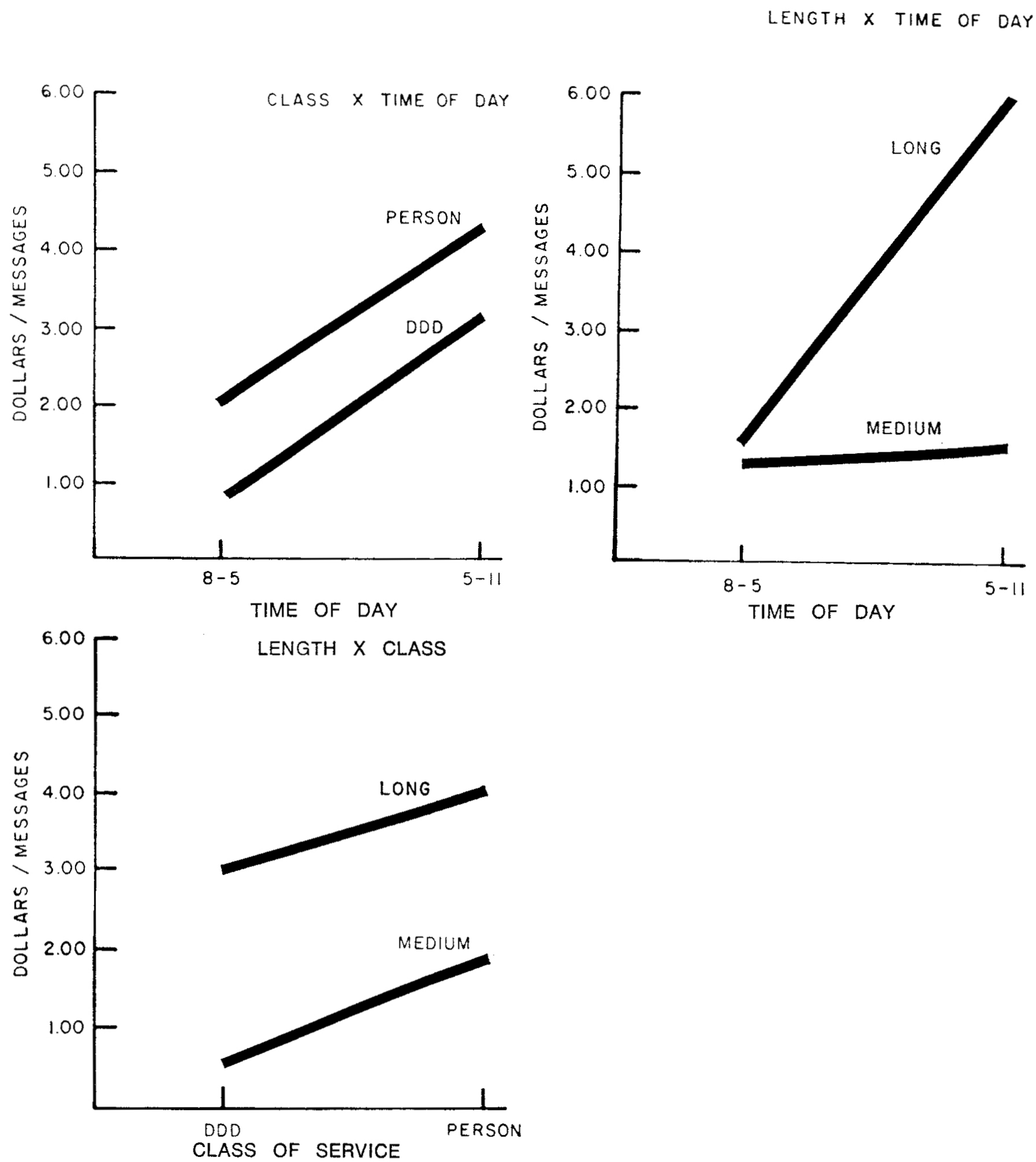


FIGURE 59

OPERATING EXPENSES / MESSAGE, (2 X 2) INTERACTIONS

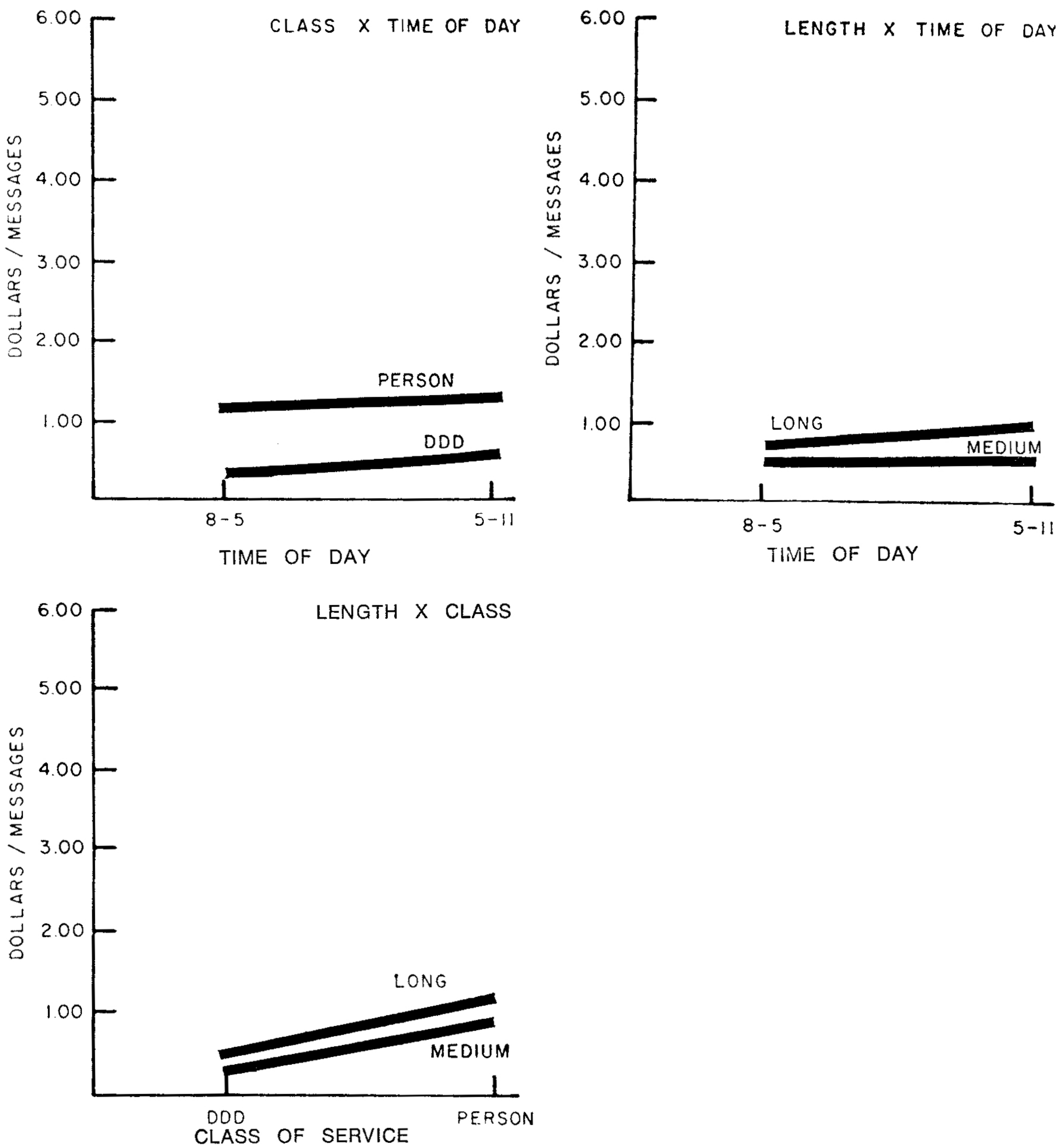
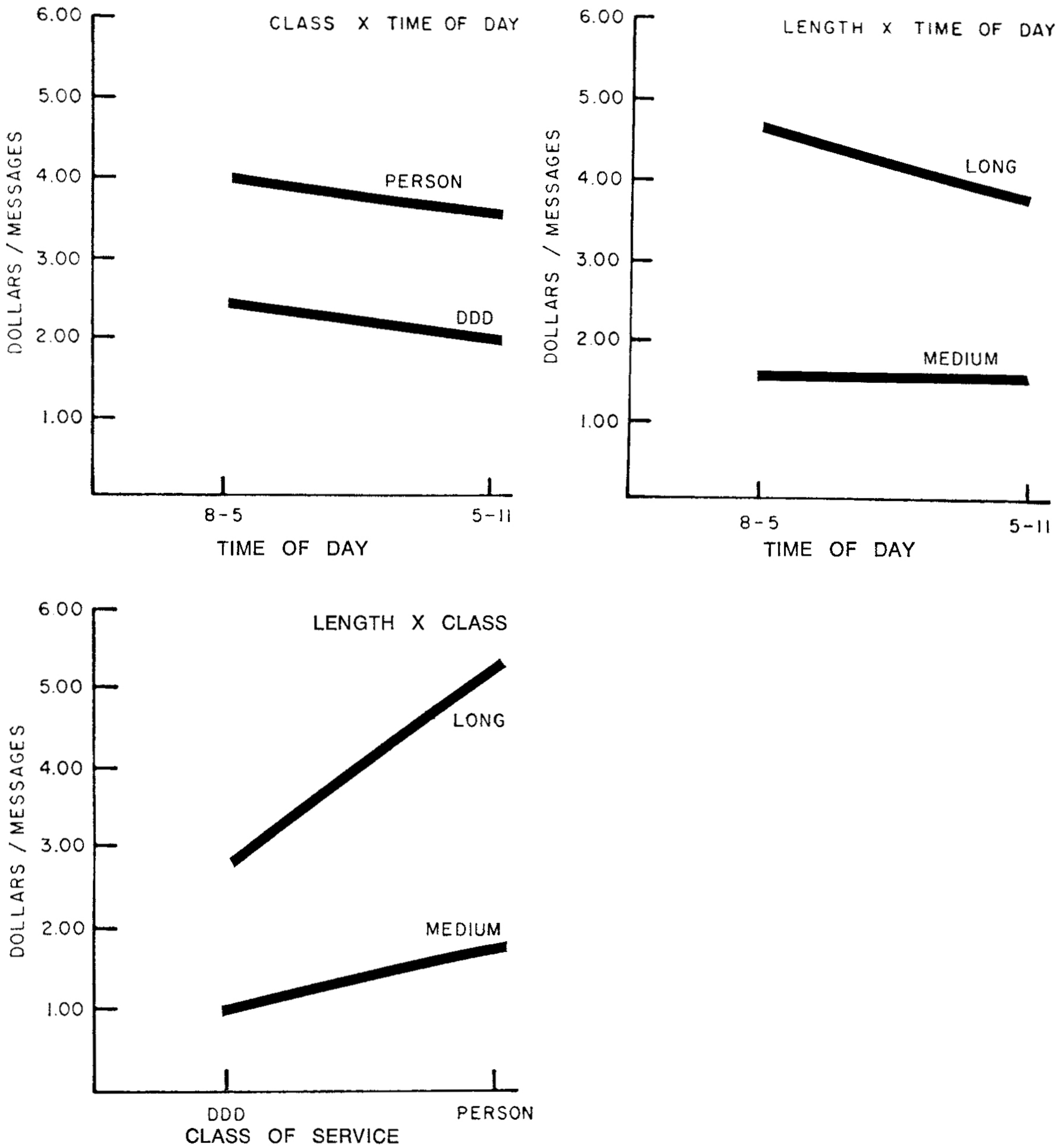


FIGURE 60

REVENUE / MESSAGE, (2 X 2) INTERACTIONS



Through study of these differing patterns, management can gain insight into the direction and degree in which prices should be varied in order to achieve specific corporate objectives relative to revenue requirements, costs, investment levels, and expense.

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